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8.EE.7.a-b

Overarching goal: Students will be able to solve multi-level equations, including no solution and infinitely many solutions. Students will understand the meanings behind the types of solutions they encounter.

Background: Students should be able to solve 1 and 2 step equations. Understand groupings and use of parenthesis to show groupings.

Included in this document there are multiple learning cycles to get to the students to solve multi-step equations. We started with a learning cycle for x's on both sides, then have another learning cycle for distributing, and finished with a learning cycle for no solution and infinitely many solutions.

We have done a few anticipating and monitoring "helps" for teachers, but you will want to continue to fill these in to meet the needs of your classroom.

Goal: Students will be able to solve equations with variables on both sides of the equation.

Developing Task: The Twins' Inheritance Part 1

Time: Should go through this learning cycle in about 1 day

      Launch: Tell the story of "The Twins' Inheritance"

      Anticipated Responses:
           Students don't need to use x's. They could use other variables

      Monitoring:
           Bring out some equation form of the first problem
           As students start writing their own equations or even as they are looking at their
           pictures, be sure they are defining their variables (attending to precision)

      Higher Level Thinking:
           Have them go back and simplify the problems to begin with

The Twins' Inheritance

Once upon a time, there were twins born to the King and Queen of the Country of Teal. The King, in all his wisdom, knew that one day the twins would grow up and inherit the kingdom with all the riches. Each year, on their birthday, the country's tax collectors would bring new gold coins to the castle for the twins. To make storage of the coins easier, the King put them into bags of equal amounts.

As the twins grew, they naturally became fond of the shiny coins they saw coming into the castle. Soon they began to wonder how many coins were in the bags. Since there were two of them, they knew that they would be sharing the riches of the kingdom. One day, upon entering the treasury, they realized that one side was marked for Princess Perfect, and the other side was marked for Prince Charming. The twins decided to see how many coins they received each year, starting with the coins they received on their first birthday. Prince Charming quickly noticed that the Princess had more bags of coins than he did.

"How unfair!" he exclaimed. "I thought it was supposed to be divided evenly!"

The princess responded, "Maybe it is equal, look at that pile of coins you have!"

On Princess Perfect's side, she counted three bulging bags and three coins. Price Charming discovers no bags on his side, but sweeps up thirty loose coins into a pile. They run to the King in disbelief!

"We thought you divided up the Kingdom's treasury evenly! You have lied to us!"

The wise King responded "I love you both exactly the same. Your treasures are of equal value. "

Can you help the twins figure out how many coins are in each bag from their 1st birthday so that the twins know the King really does love them equally?

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Each year the tax collectors bring more bags and coins into the treasury. The bags are different in size each year because the King is always beheading the seamstress. (She just can't get his

robe perfect!). Of course, the twins want to know the amount of coins in each bag…they are designing their future castles and need a budget

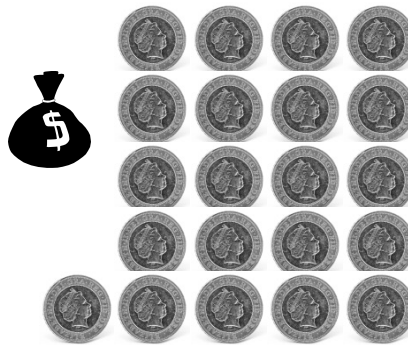The Twins' Inheritance—Part 1

Name_____ Period_____

Find the number of coins in each bag (remember, each bag is the same size, so will hold the same amount of coins). Prove that your answer works and gives the twins equal amounts of coins.

<u>Year 1</u>
Princess Perfect   **Prince Charming**

1) What is the first thing you would do to find the coin amount?

2) How many coins are in each bag? Show how you got your answer.

3) How would you apply your above strategy to solve: $4x + 5 = 8x + 1$
(If you're stuck, try answering question 4 first, then come back to this question).

4) What are the similarities and differences between question 2 and question 3?

Year 2
The coins that Prince Charming and Princess Perfect received for their 2nd birthday appear to be of different amounts yet again! Princess Perfect received 6 bags and 12 coins. Prince Charming received 3 bags and 36 coins. Find the amount in the bags to prove that Charming and Perfect received equal amounts that year.

5) Draw out the picture to match the situation.

6) What is the first thing you would do to find the coin amount?

7) How many coins are in each bag? Show how you got your answer.

8) How could you put a mathematical representation to the picture you drew in problem 5?

9) How would you apply your above strategy to solve the following problem: $7x + 23 = 12x + 8$
(If you're stuck, try answering question 10 first, then come back to this question).

10) What are the similarities and differences between question 7 and question 9?

Solidify Task: Bring the class together, solidify the ideas, whole class discussion with pre-selected students demonstrating ideas at the board.

Task: Have the students write their own "Prince and Princess Scenario" with bags on both sides of the treasury.   Then have them trade their problems with a different group and work them out.

Share 1 or 2 examples with the class.

Practice Task:
Worksheet or homework from your book (specifically with variables on one or both sides) (no distributing, yet!)

Learning Cycle 2

Goal: To use the distributive property on one side of the equation to solve multi-step equations.

Developing Task: The Twins' Inheritance Part 2

Time: Should go through this learning cycle in about 1 day

       Launch: Talk about what a "set" of things means

       Anticipated Responses:

       Monitoring:
           Help students come up with an algebraic representation for their Year 3 drawing

       Higher Level Thinking:
           Have them go back and write the same problems in a different way (without the
           distribution, or some other form)

The Twins' Inheritance—Part 2

Name_____ Period_____

<u>Year 3</u>
For the Prince and Princess' 3rd birthday, the tax collectors brought their coins as usual. The Prince received 4 sets of 1 bag and 2 coins. The Princess received 3 bags and 23 coins. Find the amount in each bag if the amount that they each received for that year is equal.

1) Draw out the picture to match the situation.

2) What is the first thing you would do to find the coin amount?

3) On the Prince's side, how many actual bags and individual coins does he have?

4) How many coins are in each bag? Show how you got your answer.

5) How would you apply your above strategy to solve the following problem: $6(2x + 7) = 13x + 30$
(If you're stuck, try answering question 6 first, then come back to this question).

6) What are the similarities and differences between question 4 and question 5?

When the Prince and Princess checked over their earnings from their 4th birthday, the Prince was excited to see that he received 3 sets of 4 bags and 7 coins. The Princess received 5 sets of 2 bags and 11 coins. Determine the amount in each bag to show that they did receive equal amounts.

7) Draw out the picture to match the situation.

8) What is the first thing you would do to find the coin amount?

9) How many coins are in each bag? Show how you got your answer.

10) How could you put a mathematical representation to the picture you drew in question 7?

11) How would you apply your above strategy to solve the following problem: $7(3x + 8) = 8(4x + 7)$
(If you're stuck, try answering question 12 first, then come back to this question).

12) What are the similarities and differences between question 9 and question 11?

Solidify Task: Bring the class together, solidify the ideas, whole class discussion with pre-selected students demonstrating ideas at the board.

Task: Have the students write their own equations that involve distribution on one or both sides.   Then have them trade their problems with a different group and work them out. These are a little harder to write, and answers might not be nice whole numbers. Could lead to a good discussion that the variable may not always be a whole number.

Share 1 or 2 examples with the class.

Practice Task:
Worksheet or homework from your book (specifically with distributing).
 (Look at www.kutasoftware.com if you need an assignment)

Goal: To make sense of equations with different types of solutions (x=a, a=a, and a=b)

Developing Task: The Twins' Inheritance Part 3

Time: Should go through this learning cycle in about 1 day

    Launch: Talk about different types of "problems" you've had in real life. Was there ever a situation where there was no good answer ("Bewitched" scene where "there is no solution."). Or was there a situation where there were lots of possible good solutions. Ex. My husband had to decide where to go to law school. He had been accepted to several different schools, all which would have been good answers to the "where to go to law school" problem.) Lead into that math is the same way…sometimes there is one answers, or maybe there could be lots of answers, or no answers.

    Anticipated Responses:

    Monitoring:
        If you notice that they wrote down "0" for their answer, have them check their answers with 0 coins in to see if the equation is true.

Name_____ Period_____

<u>Year 5</u>

For their 5th birthday, the Prince received 2 sets of 3 bags and 4 coins. The Princess received 6 bags and 5 coins. Determine the number of coins in each bag.

1) Draw the picture to represent this situation

2) Go through the process to figure out how many coins are in each bag. Do you notice anything different about this problem than you have in previous days?

3) Explain what is going on in this situation.

4) Obviously, the Treasurer has messed something up. If you were the King, what would you do to punish the treasurer?

For their 6th birthday, the Prince received 6 bags and 9 coins. The Princess received 3 sets of 2 bags and 3 coins. Determine the number of coins in each bag.

5) Draw the picture to represent this situation

6) Go through the process to figure out how many coins are in each bag. Do you notice anything different about this problem than you have in previous days?

7) Explain what is going on in this situation.

8) Did the Treasurer mess up this time? How do you know?

9) Given the equation $4(5x + 7) = 20x + 28$, is x=2 a possible solution? Is x=-3 a possible solution?

10) Why do multiple solutions work for the above equation?

Solidify Task: Make sure your discussion centers around the wide variety of "solutions" for Year 6. Stress the difference between no solution and all real numbers.

Task: Have students write 3 different equations. One equation that has 1 solution, one equation that has no solution, and one equation that has many solutions. Have them "blindly" trade papers with three different groups and solve the new problems. Then they will have to identify the different types of solutions.

Share 1 or 2 examples with the class.

Practice Task:
Worksheet or homework from your book.
 (Look at www.kutasoftware.com if you need an assignment)

Higher Level Thinking:

Have them go back and write the same problems in a different way (without the distribution, or some other form)