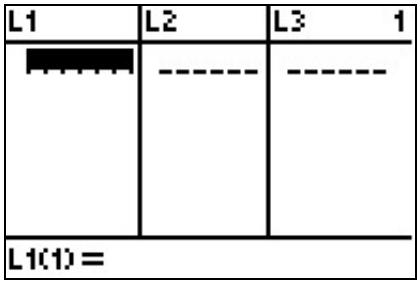
Problem 1 – The first constraint

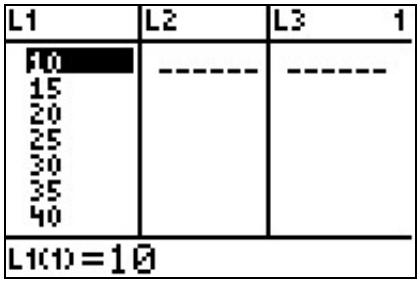
In this problem, you will build a model of a real-life situation by writing linear inequalities to represent the constraints on the situation. You will see how the set of solutions changes as each constraint is added.

The owner of a birdhouse business can make a birdhouse in 90 minutes. He can work at most 40 hours a week making birdhouses. Write an inequality to represent the number of birdhouses he can make in a week, *x*, given this constraint.

We can use the calculator to view the solution set to this inequality in two different ways. The first is with the **Lists** feature. Press [STAT] then [ENTER] to open the **List Editor**. Clear any data from $\mathbf{L_1}$ and $\mathbf{L_2}$.



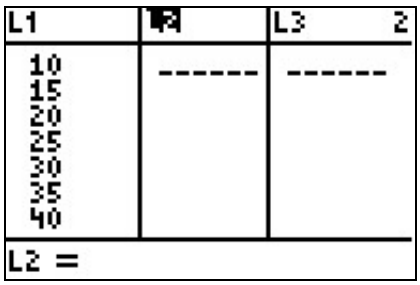
Enter a range of values of *x* in $\mathbf{L_1}$. These values represent different numbers of birdhouses that the owner could make in a week.



Arrow up to the top of $\mathbf{L_2}$ and type your inequality. Replace *x* with $\mathbf{L_1}$.

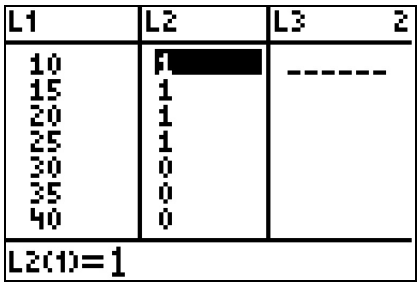
To type $\mathbf{L_1}$, press [2nd] + [1].

Inequality symbols are found in the **Test** menu. ([2nd] + [MATH]).

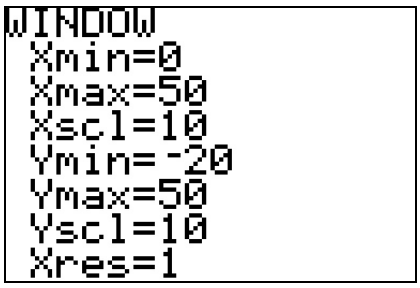


A value of 1 in $\mathbf{L_2}$ means that the inequality is true for the value of *x* in that row. A value of 0 means that the inequality is false for the value of *x* in that row. Each *x* value in $\mathbf{L_1}$ with a value of 1 in $\mathbf{L_2}$ is a solution to the inequality.

1. Can the owner make 10 birdhouses in a week? 20? 30?



Another way to view the solutions to this inequality is by graphing. First, adjust the window settings as shown.



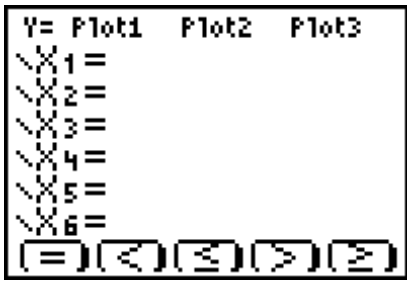
Press APPS to open the **Applications** menu and choose **Inequalz** to open the **Inequality Graphing** application**.**

Press any key to begin.

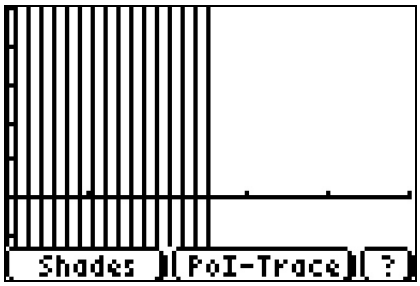
To graph an inequality with the variable *x*, move to the text **X=** in the upper left corner of the screen and press ENTER. This changes all the equations from the form **Y=** to **X=**.

Solve your inequality for *x* and enter it in $\mathbf{X_1}$. Round any decimals to the nearest hundredth. To change the = to the correct inequality symbol, press ALPHA and the function key that corresponds to the symbol you want.

For example, to make ≤, press ALPHA + ZOOM.



Press GRAPH to view the graph. All of the points in the shaded area represent solutions to the inequality.



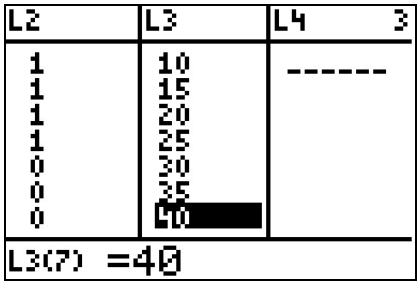
Problem 2 – Another constraint

The owner decides to hire a expert carpenter to help make the birdhouses. The expert carpenter can make a birdhouse in 75 minutes. However, the owner can only afford to pay the expert for 20 hours of work per week.

Write an inequality to represent the number of birdhouses the expert can make in a week, *y*, given this constraint.

Press [STAT] then [ENTER] to open the **List Editor**. Clear any data from L_3 and L_4.

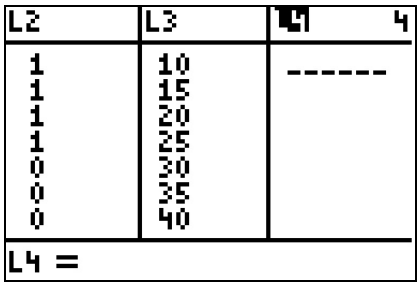
Enter a range of values of y in L_3. These values represent different numbers of birdhouses that the expert carpenter could make in a week.



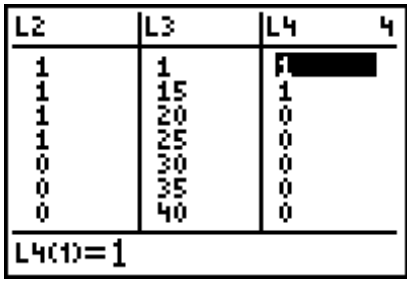
Arrow up to the top of L_4 and type your inequality. Replace y with L_3.

To type L_3, press [2nd] + [3].

Inequality symbols are found in the **Test** menu. ([2nd] + [MATH]).



A value of 1 in L_4 means that the inequality is true for the value of y in that row. A value of 0 means that the inequality is false for the value of y in that row. Each y value in L_3 with a value of 1 in L_4 is a solution to the inequality.



2. Can the expert make 10 birdhouses in a week? 20? 30?

Look for the rows that have a 1 in L_2 and a 1 in L_4. The x- and y-pairs from these rows are solutions to **both** inequalities.

For example, if $L_1 = 10$, and $L_3 = 10$, and there is a 1 in L_2 and a 1 in L_4, the ordered pair (10, 10) is a solution to the system

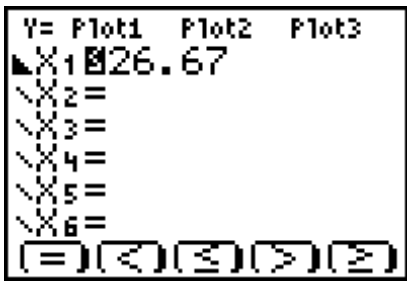
$$\begin{cases} 1.5x \le 40 \\ 1.25y \le 20 \end{cases}.$$

In fact, any combination of an x-value that is a solution to the first inequality and a y-value that is a solution to the second inequality is a solution to this system, even if the two values are not in the same row. So (10, 15) and (20, 10) are also solutions.

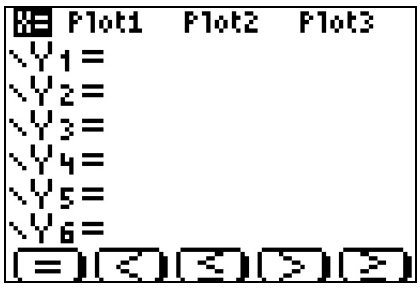
3. What does the solution (10, 15) represent in this situation?

4. List as many solutions to the system as you can.

Another way to view the solutions to this system is to graph the two inequalities together. Press [Y=] to return to the **Inequality Grapher**.

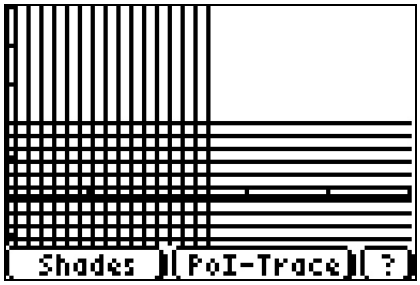


To graph an inequality with the variable *y*, move to the text **Y=** in the upper left corner of the screen and press ENTER. This changes all the equations from the form **X=** to **Y=**.

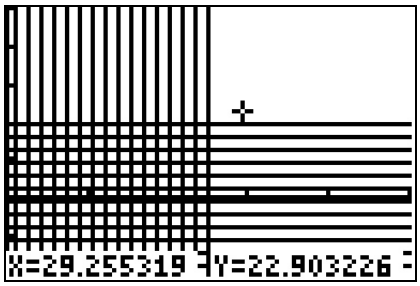


Solve your inequality for *y* and enter it in $\mathbf{Y_1}$. Remember to change the = to the correct inequality symbol.

Press GRAPH to view the graph. All of the points in the horizontally striped area are solutions to the first inequality. All the points in the vertically striped area are solutions to the second inequality. The points where these two areas overlap are solutions to the both inequalities, and hence solutions to the system.



Use the arrow keys to move the cursor to the intersection of the two shaded areas.



5. List several points that are within this area.

6. Compare your answer to Question 5 with your answer to Question 4.

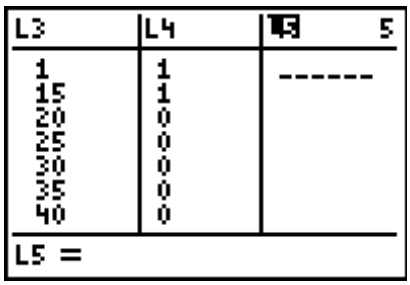
Problem 3 – A final constraint

A store would like to place an order for 50 birdhouses a week. Can the owner and the expert working together fill the order?

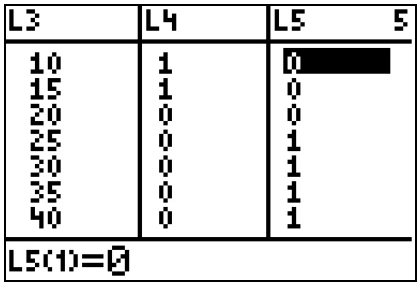
Write an inequality to represent the number of birdhouses the owner would have to make each week (*x*) and the expert would have to make each week (*y*) to fill this order.

Press STAT then ENTER to open the **List Editor**. Clear any data from $\mathbf{L_5}$.

Arrow up to the top of $\mathbf{L_5}$ and type your inequality. Replace *x* with $\mathbf{L_1}$ and *y* with $\mathbf{L_3}$.



A value of 1 in $\mathbf{L_5}$ means that the inequality is true for the values of x and y in that row. A value of 0 means that the inequality is false for the values of x and y in that row. Each (x, y) pair with a value of 1 in $\mathbf{L_5}$ is a solution to the inequality.



7. List several solutions to this inequality.

Look for rows that have a 1 in $\mathbf{L_2}$, a 1 in $\mathbf{L_4}$, and a 1 in $\mathbf{L_5}$. The x and y pairs from these rows are solutions to **all three** inequalities, in other words, solutions to the system

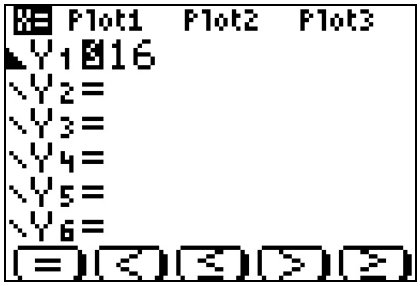
$$\begin{cases} 1.5 \le 40 \\ 1.25y \le 20 \\ x + y \ge 50 \end{cases}.$$

8. Are there any such rows? List as many solutions to the system as you can.

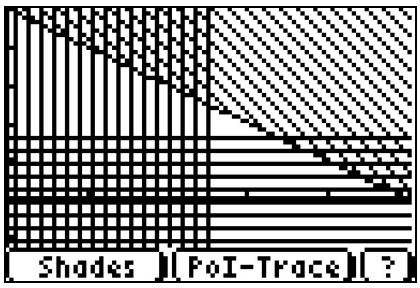
9. What does your answer to question 8 mean in this situation?

To view the solutions to this system, graph all three inequalities together. Press [Y=] to return to the **Inequality Grapher**.

Solve your inequality for y and enter it in $\mathbf{Y_2}$. (You could also solve the inequality for x and enter it into $\mathbf{X_2}$ with the same result.)



Press [GRAPH] to view the graph. All of the points in the horizontally striped area are solutions to the first inequality. All the points in the vertically striped area are solutions to the second inequality. All the points in the diagonally striped area are solutions to the third inequality.



10. Is there an area where all three of these overlap?

11. What does this mean about the solutions to this system?

Challenge

12. Use the **Points of Interest – Trace** feature of the **Inequality Grapher** to find the maximum number of birdhouses that the owner and the expert can make in a week. (Press [ALPHA] + [ZOOM] to access this feature and use the arrow keys to move between the points of interest on the graph.)