

Exponentialis ~ Logarithmus

by Suzanne Moyers

Time required45-55 minutes

Activity Overview

In this story-style activity, students work through a step-by-step review of solving exponential equations using logarithms. At first, they are guided through process of using logarithms and checking them, with the help of "Terry Plotter the mathemagician". Then, students review identities and properties of logarithms, with trial examples of each. The objective of the activity is to connect exponential equations with their logarithmic counterparts, working with a variety of bases.

Concepts

- Solving exponential equations using logarithmic forms of the equations.
 - Review of properties of logarithms, working with examples of each property.
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Teacher Preparation

This activity is designed for use in the Pre-Calculus, Advanced Mathematics, or Algebra 2 classroom. It uses a conversational narrative to help students "lighten up" about solving exponential equations using logarithmic form.

Prior to the start of this activity, students should have been introduced to exponential and logarithmic forms. The activity may be used to introduce properties of logarithms, or as a review of properties.

- Each exponential equation applies logarithmic form for its solution, using the ability of the TI-Nspire to solve a logarithm for any base.
 - Screenshots on the following pages include frames from the **ExpLog.tns** file, before it has been filled in by the student, AND frames from the **ExpLogAns.tns** file, which is completed as the student's final document.
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Classroom Management

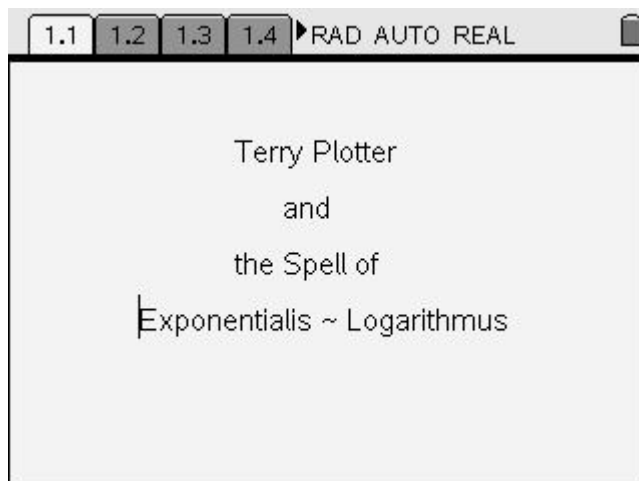
- This activity is intended to be completed **individually**, as a review prior to assessment or as a segue into in-depth treatment of the properties of logarithms.
 - Prior to the activity, load the **ExpLog.tns** file onto class handhelds to be used by students and teacher.
 - Students should be directed **NOT TO SAVE** the document during or after the activity, if it is to be used as a review in subsequent classes.
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TI-Nspire™ Applications

Calculator, Notes

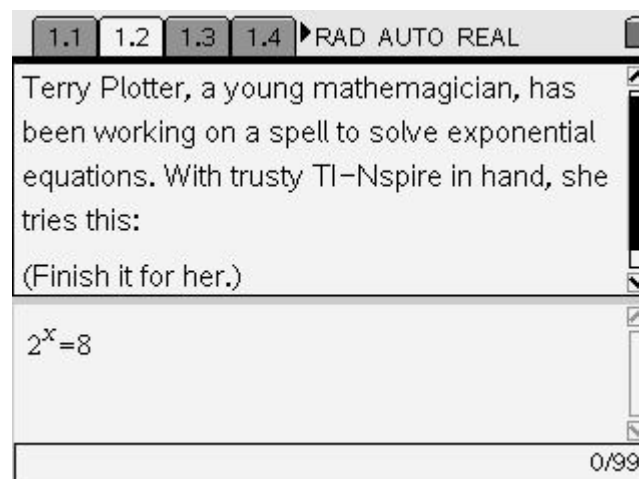
Pre-Calculus *TI-Nspire™* Exponentials & Logarithms

Teacher Info: Today we will review exponential equations and logarithmic forms, with the help our fictional friend and “mathemagician”, Terry Plotter. She is working on a new spell, called the “Exponentialis ~ Logarithmus”, to help her solve exponential equations using logarithmic forms. With the TI-Nspire, we will also review identities and properties of logarithms.

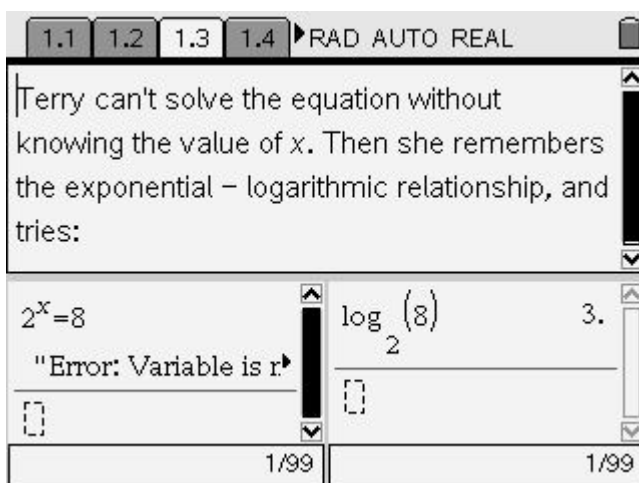


Problem Set 1 – Guided Problem Solving

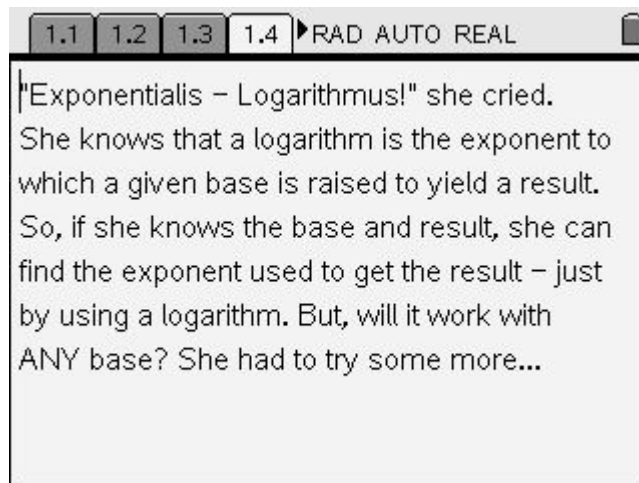
Students begin by solving exponential equations using logarithms, using pre-filled Calculator panes in the TI-Nspire file. They must Ctrl-Tab to the problem pane, place the cursor at the right end of the equation, then hit ENTER. They'll repeat these steps with each equation.



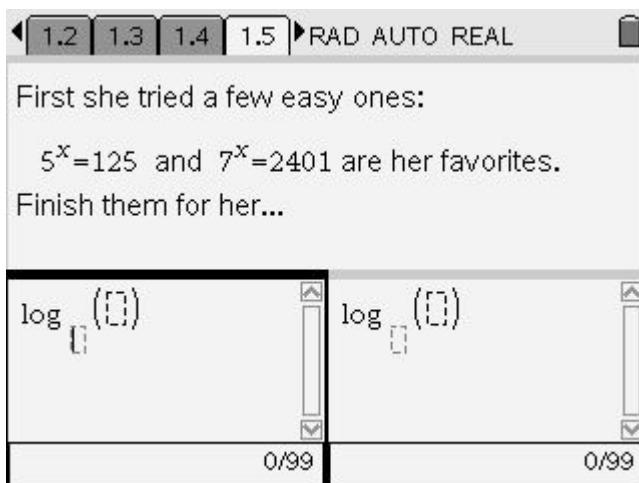
On page 1.3, students are shown why logarithmic form must be used to solve the exponential equation. They are also shown the result of the logarithmic form of the equation.



On page 1.4, dialogue-style narrative from Terry explains the use of logarithmic form and its rationale. The question is posed as to whether the process can be done with ANY base.

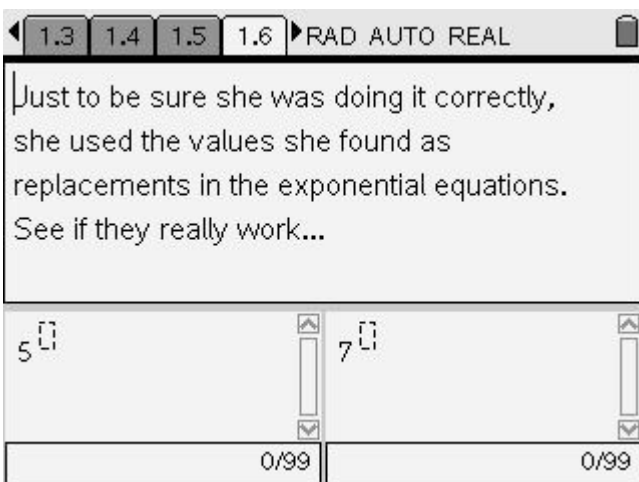


On page 1.5, students are prompted to solve two exponentials using logarithmic form. The logarithmic expressions are already set up. Students must use Ctrl-Tab to move from pane to pane. Within a pane, Tab moves within the log expression, from the base up. They must hit enter to evaluate the expression.



On page 1.6, students are directed to check the exponents found on page 1.5, this time using exponential form.

Throughout subsequent pages in the document, they are reminded to check each logarithmic form using the exponential equation.

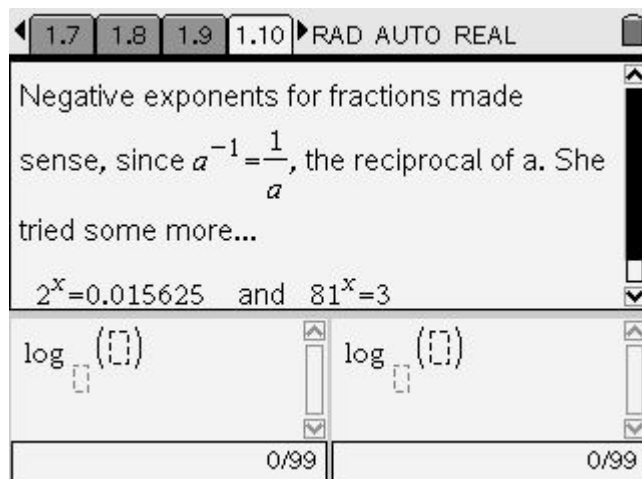


On page 1.7, students are solving equations independently, with reminders to check their work.

On pages 1.8 and 1.9, exponential equations involving fractions and decimals are solved using logarithmic forms.

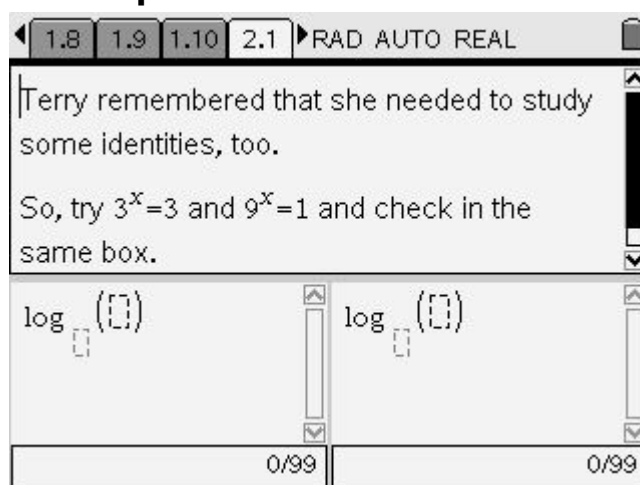
Checking using exponential form is done on each page.

Problem Set 1 ends with a review definition of negative exponents and two more problems. By now, students should be checking each result in the same Calculator pane as the problem is worked.

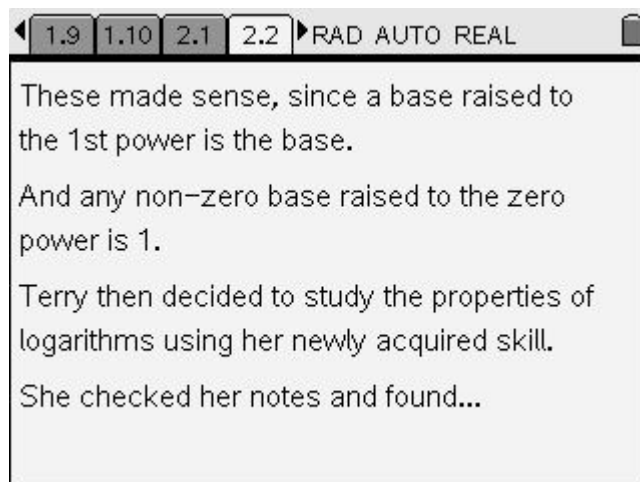


Problem Set 2 – Logarithmic Identities and Properties

In this set, students review logarithmic identities and properties. Identity examples are illustrated on page 2.1



Page 2.2 reviews the exponential definitions that accompany logarithmic identities.



On page 2.3, students review the Product Property, with examples to work and check.

The screenshot shows a TI-Nspire calculator interface. At the top, a navigation bar includes tabs for 1.10, 2.1, 2.2, and 2.3, with 2.3 selected. The title bar reads 'RAD AUTO REAL'. The main display area is titled 'The Product Property' and contains the text: 'To expand a logarithmic product, we need the sum of the logs of the factors. Check to see if they match...'. Below the text, two mathematical expressions are shown side-by-side: $\log_2(32)$ on the left and $\log_2(4) + \log_2(8)$ on the right. At the bottom of the screen, there are two input fields, each containing '0/99'.

Page 2.4 reviews the Quotient Property, with examples to work and check.

The screenshot shows a TI-Nspire calculator interface. At the top, a navigation bar includes tabs for 2.1, 2.2, 2.3, and 2.4, with 2.4 selected. The title bar reads 'RAD AUTO REAL'. The main display area is titled 'The Quotient Property' and contains the text: 'The log of a quotient is difference of the logs of the numerator and denominator. Check this one out...'. Below the text, two mathematical expressions are shown side-by-side: $\log_5\left(\frac{1}{125}\right)$ on the left and $\log_5(1) - \log_5(125)$ on the right. At the bottom of the screen, there are two input fields, each containing '0/99'.

On page 2.5, students review the Power Property, by working and checking more examples.

The screenshot shows a TI-Nspire calculator interface. At the top, a navigation bar includes tabs for 2.2, 2.3, 2.4, and 2.5, with 2.5 selected. The title bar reads 'RAD AUTO REAL'. The main display area is titled 'The Power Property' and contains the text: 'Terry thought this might be harder. But, it's not! Check using exponents to be sure!'. Below the text, two mathematical expressions are shown side-by-side: $\log_3(9^2)$ on the left and $2\log_3(9)$ on the right. At the bottom of the screen, there are two input fields, each containing '0/99'.

Page 2.6 illustrates combined Quotient and Power Properties, with guided examples.

2.3 2.4 2.5 2.6 RAD AUTO REAL

Terry decided to combine some properties, just to make things interesting...
Finish this one and check.

$$3\log_2(4) - 2\log_2(4)$$

$$\log_2\left(\frac{4}{16}\right)$$

0/99 0/99

On page 2.7, students must extend their ability to solve exponential equations logarithmically, since the exponent is $2x$ rather than simply x .

2.4 2.5 2.6 2.7 RAD AUTO REAL

Terry just had to try one more, before starting her mathmagic homework.
Complete this one to see how it works out:
 $5^{2x} = 625$

$$\log_{[]}([])$$

$$\log_{[]}([])$$

0/99 0/99

On the final page, 2.8, a light-hearted narrative lets students know they are finished.

Remind students NOT TO SAVE the document. If they SAVE, their work will overwrite the original ExpLog.tns document. The original (without student work) will have to be reloaded to the handheld.

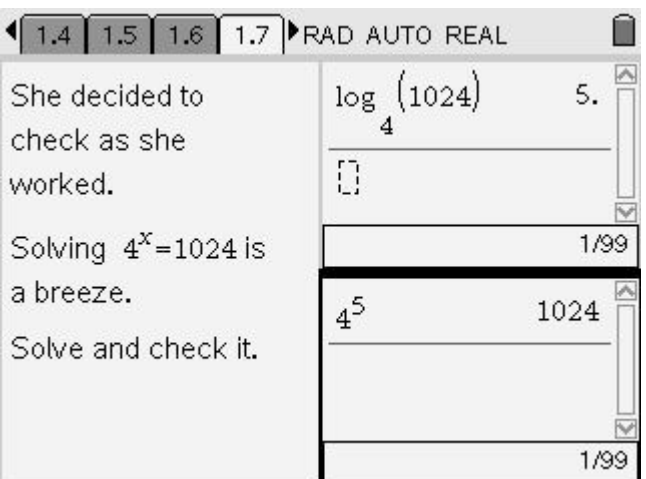
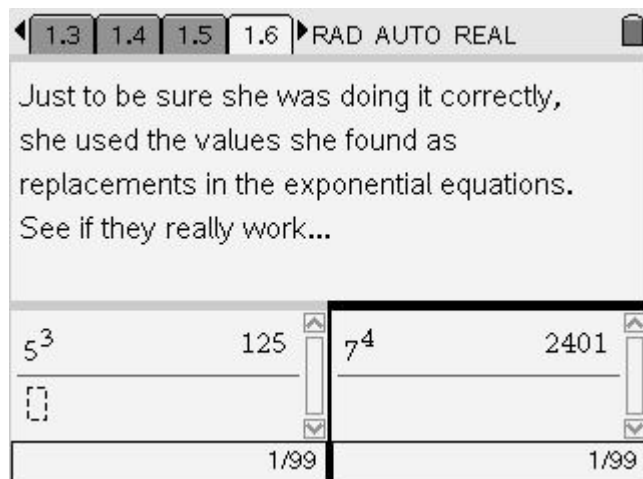
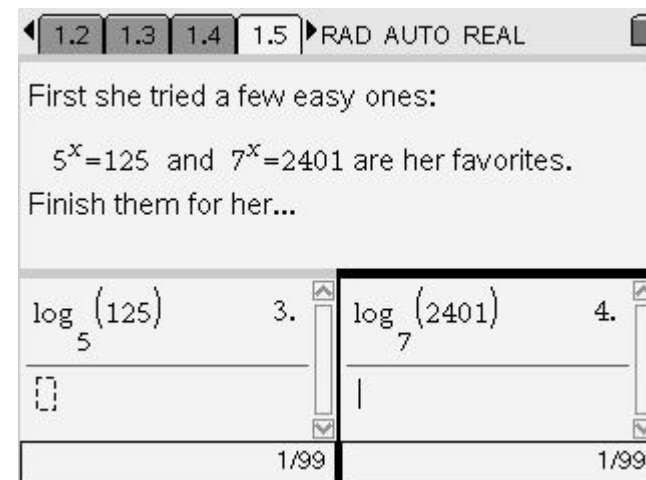
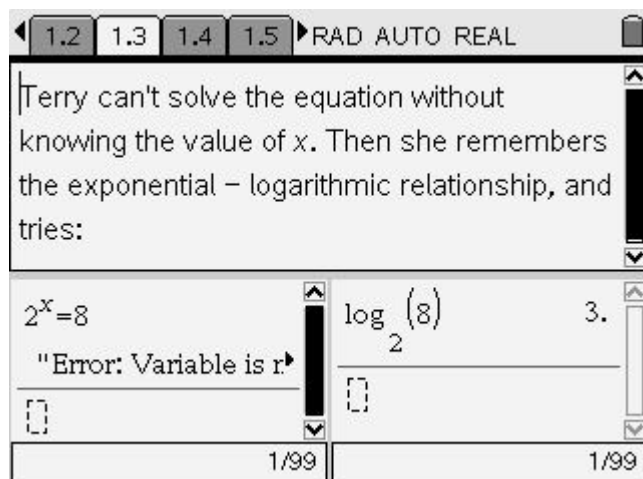
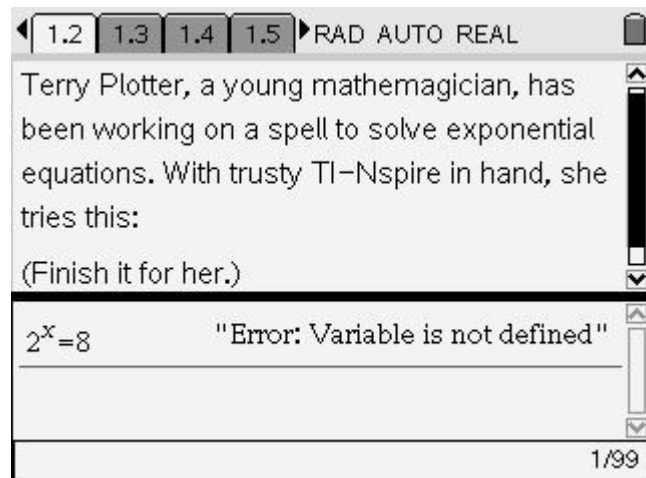
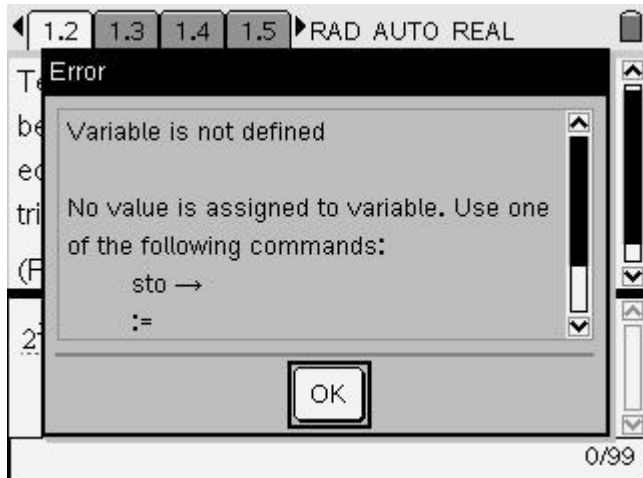
2.5 2.6 2.7 2.8 RAD AUTO REAL

Back at her dorm at Logwarts, (House of Graphindor, of course), Terry is ready to start her homework to review for tomorrow's test.
She has mastered the Spell of Exponentialis ~ Logarithmus!

ExpLogAns.tns

Completed Student Document

(page 1)



ExpLogAns.tns

Completed Student Document

(page 2)

1.6 1.7 1.8 1.9 RAD AUTO REAL

$\log_6\left(\frac{1}{7776}\right)$ -5.

Next, she tried it with fractions.

Solve $6^x = \frac{1}{7776}$

and check.

$6^{-5} = \frac{1}{7776}$

1/99

1.6 1.7 1.8 1.9 RAD AUTO REAL

Terry tried working with decimals, too.

Solve $10^x = 0.000000001$

and check.

$\log_{10}(1.E-9)$ -9.

$\frac{1}{1000000000}$

1/99

1.8 1.9 1.10 2.1 RAD AUTO REAL

Negative exponents for fractions made sense, since $a^{-1} = \frac{1}{a}$, the reciprocal of a. She tried some more...

$2^x = 0.015625$ and $81^x = 3$

$\log_2(.015625)$ -6.

$\log_{81}(3)$.25

1/99

1.8 1.9 1.10 2.1 RAD AUTO REAL

Terry remembered that she needed to study some identities, too.

So, try $3^x = 3$ and $9^x = 1$ and check in the same box.

$\log_3(3)$ 1.

$\log_9(1)$ 0.

1/99

1.10 2.1 2.2 2.3 RAD AUTO REAL

The Product Property

To expand a logarithmic product, we need the sum of the logs of the factors. Check to see if they match...

$\log_2(32)$ 5.

$\log_2(4) + \log_2(8)$ 5.

1/99

2.1 2.2 2.3 2.4 RAD AUTO REAL

The Quotient Property

The log of a quotient is difference of the logs of the numerator and denominator.

Check this one out...

$\log_5\left(\frac{1}{125}\right)$ -3.

$\log_5(1) - \log_5(125)$ -3.

1/1

ExpLogAns.tns

Completed Student Document

(page 3)

2.2 2.3 2.4 2.5 RAD AUTO REAL

The Power Property

Terry thought this might be harder. But, it's not!

Check using exponents to be sure!

$\log_3(9^2)$	4.	$2 \cdot \log_3(9)$	4.
1/99		1/99	

2.3 2.4 2.5 2.6 RAD AUTO REAL

Terry decided to combine some properties, just to make things interesting...

Finish this one and check.

$3 \cdot \log_2(4) - 2 \cdot \log_2(4)$	2.	$\log_2\left(\frac{4^3}{16}\right)$	2.
1/1		1/99	

2.5 2.6 2.7 2.8 RAD AUTO REAL

Terry just had to try one more, before starting her mathemagic homework.

Complete this one to see how it works out:

$5^{2x} = 625$

$\log_5(625)$	4.	5^4	625
1/99		1/99	

2.5 2.6 2.7 2.8 RAD AUTO REAL

Back at her dorm at Logwarts, (House of Graphindor, of course), Terry is ready to start her homework to review for tomorrow's test.

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