Performance Assessment Task

Functions
Grade 9

The task challenges a student to demonstrate understanding of the concepts of relations and functions. A student must be able to understand functions and select, convert flexibly among, and use various representations for them. A student must be able to identify linear points on a coordinate grid and name them. A student must determine the equation for a linear function from a graph or from coordinates. A student must be able to recognize non-linear points that form a parabola and estimate the graph of the curve. A student must be able to find the equation for a parabola given some of the coordinate points. A student must be able to distinguish between the features of a linear, quadratic and exponential graph and their equations.

Common Core State Standards Math - Content Standards

High School – Functions – Interpreting Functions
Analyze functions using different representations.
F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

High School – Functions – Linear, Quadratic, and Exponential Models
Construct and compare linear, quadratic, and exponential models and solve problems.
F-LE.3 Observe using graphs and tables that a quantity inkling exponentially eventually exceeds a quantity increasing linearly, quadratically or more generally as a polynomial function..

High School – Algebra – Creating Equations
Create equations that describe numbers or relationships
A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinates axes with labels and scales.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Assessment Results				
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
9	2008	8	4	26 %

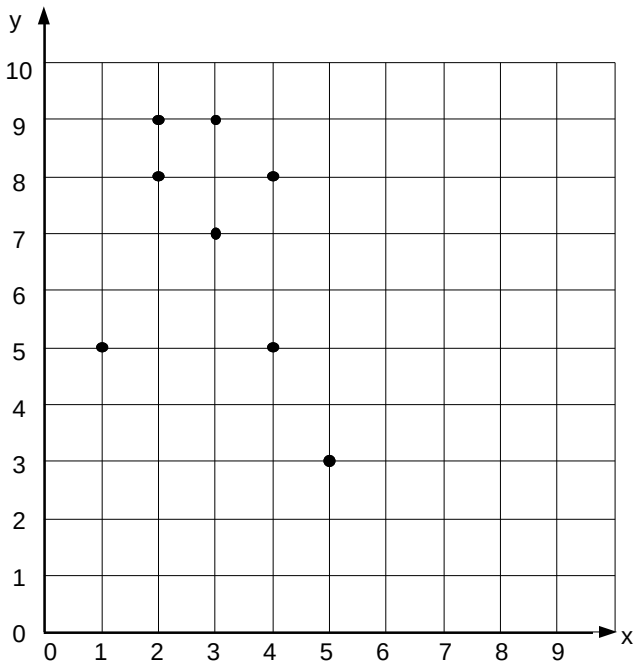
Functions

This problem gives you the chance to:
• work with graphs and equations of linear and non-linear functions

On the grid are eight points from two different functions.

• four points fit a **linear** function

• the other four points fit a **non-linear** function.



For the **linear** function:

1.Write the coordinate pairs of its four points.

_____

_____

_____

_____

Draw the line on the grid.

2.Write an equation for the function.
Show your work.

_____

For the **non-linear** function:

3. Write the coordinate pairs of its four points.

_____ _____ _____
_____

Draw the graph of the function on the grid.

4.

The non-linear function is quadratic

The non-linear function is exponential

Chris **Alex**

Who is correct? _____

Explain your reasons.

_____

_____

_____

_____

_____

_____

5. Write an equation that fits the non-linear function.

_____

Show your work.

Functions	Rubric	
The core elements of performance required by this task are: • work with graphs and equations of linear and non-linear functions Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: **(2, 9), (3, 7), (4, 5), (5, 3)** **and** Draws a correct line on the grid.	1	1
2. Gives correct answer: **$y = 13 - 2x$**	2	2
3. Gives correct answers: **(1, 5), (2, 8), (3, 9), (4, 8)** Draws a correct curved graph or equivalent	1 1	2
4. Gives correct answer: **Chris** **and** Gives a correct explanation such as: The graph has a turning point. **or** It is part of a parabola.	1	1
5. Gives correct answer: $y = \mathbf{6x - x^2}$ or equivalent such as $-(x-3)^2 + 9$ Shows some correct work such as: Substitutes coordinates in $y = ax^2 + bx + c$	1 1	2
Total Points		**8**

Functions

Work the task and look at the rubric. What strategies or big mathematical ideas might students use to help them find the formulas for part 2 and 3?

Look at student work for part one. How many of them were able to pick the correct points? _____
How many put (2,8) instead of (2,9)? _____ What other misconceptions or errors did you notice for this part?

Now look at student work for part 2. How many of your students:

y= 13 – 2x	y = mx+b	An expression with – 2x, incorrect constant	Numerical expression, no variable	Table or list of points	Other

What does each answer show about that students understand about equations? Variables? Checking more than one set of points? Connecting algebraic and graphic representations?
How many of your students were unwilling to attempt an equation?_____
Make a list of the types of strategies students used to solve this part of the task.

Were there strategies that you might have expected them to use that you didn't see? Which strategies most consistently gave the correct answers?

Now look at the graphs for part 3. How many of your students selected the correct 4 points?_____
How many could visualize the points as a parabola?_____ How many connected the points in a pointy shape?_____ How many tried to make a quadrilateral?_____ How many made a square shape?_____
What experiences have your students had with graphing parabolas? Do students get an opportunity to explore how different parts of the equation effect the shape of the graph?

Now look at the explanations for quadratic and exponential equations. How many of your students could give:

- a reasonable explanation?_____
- were unwilling to answer?_____
- thought exponential meant exponents in the equation?_____
- thought quadratic meant 4 points or a 4-sided figure?_____

- the graph isn't square, so it can't have square numbers?_____
- thought both answers were correct?_____

How do we help students develop academic language? How often do students have discussions in class where they need to use these words to help express their ideas? How do we help orchestrate or foster their experiences?

Finally look at student work for part 5. How many of your students put:

- $y = 6x - x^2$? _____
- an equation with x^2? _____
- a linear equation, that would fit one of the points? _____
- attempted to use a table? _____
- tried $ax^2 + bx + c$, but couldn't get to a final answer? _____
- a numerical expression? _____
- no attempt? _____
- other? _____

What thinking did you see, whether successful or not, that could be built upon in future lessons?

In what way could you use some partial student work to push students to do more investigation around this idea? Were there any problem solving strategies that were worth talking about?

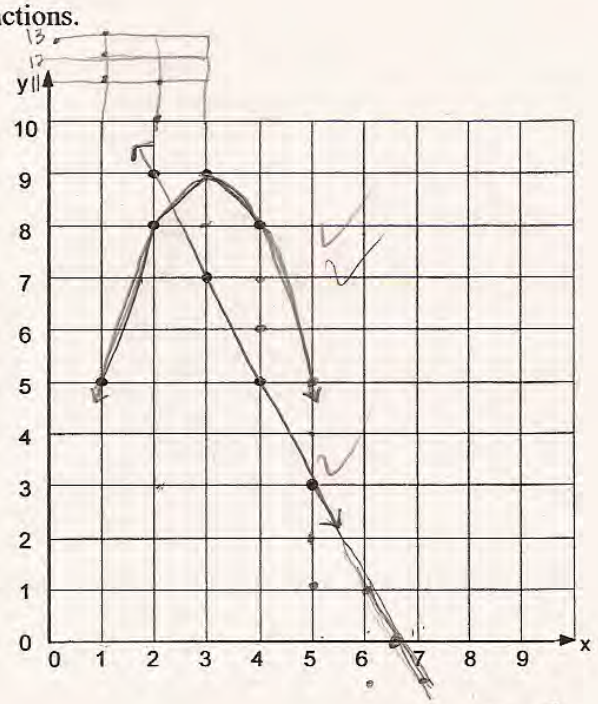
Looking at Student Work on Functions

Student A is able to think about the equation of a line and use it to find the equation in part 2. Notice how the student calculates the slope and then extends the graph to think about the y-intercept. In part 4 the student has the language to talk about quadratic functions. Notice that the student has added an additional point on the graph to make the parabola symmetrical. In part 5 the student uses the formula for the turning point of a parabola to check that the equation is correct.

Student A

On the grid are eight points from two different functions.

- four points fit a **linear** function
- the other four points fit a **non-linear** function.



For the **linear** function:

1. Write the coordinate pairs of its four points.

(5,3) ✓
(4,5) ✓
(3,7) ✓
(2,9) ✓

Draw the line on the grid.

2. Write an equation for the function.
Show your work.

$y = mx + b$
up 2 over −1
$m = \frac{2}{-1}$
$y = -2x + 13$ ✓ ✓

For the **non-linear** function:

3. Write the coordinate pairs of its four points.

(1,5) ✓ (2,8) ✓ (3,9) ✓ (4,8) ✓ ✓

Student A, part 2

Chris **Alex**

Who is correct? Chris

Explain your reasons.

A quadratic formula is the equation for a parabla. The non-linear function is the shape of a parabla. If you find another coordinate, it will continue the parabla shape, making the function quadratic.

5. Write an equation that fits the non-linear function. Show your work.

$-9+18=9$

$y=-x^2+6x$

(3,9)

$\frac{-b}{a(2)}$ (3,9) vertex $\frac{-6}{-2}=3$

Student B shows an interesting thought process, starting with a table to look at differences to find the turning point. The student thinks about the direction of the parabola. So, using a variety of pieces of knowledge the student is able to put together a correct equation. Notice that the student checks the equation against all the given points.

Student B

4.

The non-linear function is quadratic.

The non-linear function is exponential.

Chris **Alex**

Who is correct? both X

Explain your reasons.

It could be exponential, because if given other points, the line could be like  . And I don't know what quadratic means, but "quad-" means 4, and there are 4 points on the graph.

5. Write an equation that fits the non-linear function. Show your work. $y = 6x - x^2$ ✓

x	1	2	3	4
y	5	8	9	8

+3 +1 −1

negative? ✓ parabola?

x-intercept = (6,0) ? 6x?

y-intercept = (0,0) ?

$y = x^2$ something

$y = 6x - x^2$ definitely

$y = 6x - x^2$

$y = 6(3) - 3^2$

$y = 18 - 9$

$y = 9$

$y = 6x - x^2$

$y = 6(2) - 2^2$

$y = 12 - 4$

$y = 8$

$y = 6(4) - 4^2$

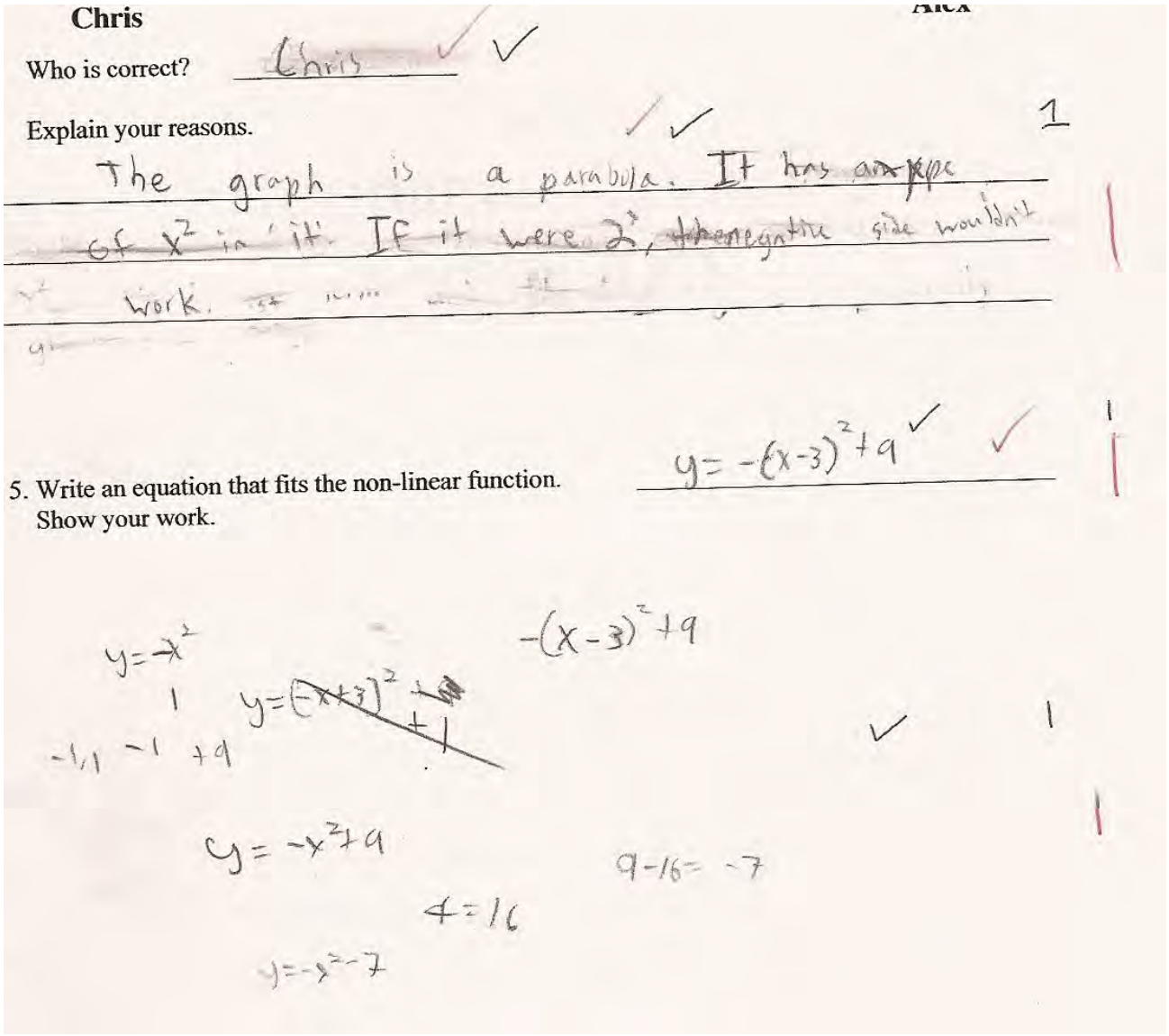
$y = 24 - 16$

$y = 8$

5

Student C comes up with an alternative equation for the parabola. *What clues can you find about the student's though process?*

Student C



Chris

Who is correct? Chris

Explain your reasons.

The graph is a parabola. It has an x^2 in it. If it were 2^2, the negative side wouldn't work.

5. Write an equation that fits the non-linear function. Show your work.

$y = -(x-3)^2 + 9$

$y = -x^2$

$y = (-x+3)^2 + 1$

$-1, 1 \quad -1 \quad +9$

$y = -x^2 + 9$

$4 = 16$

$y = -x^2 - 7$

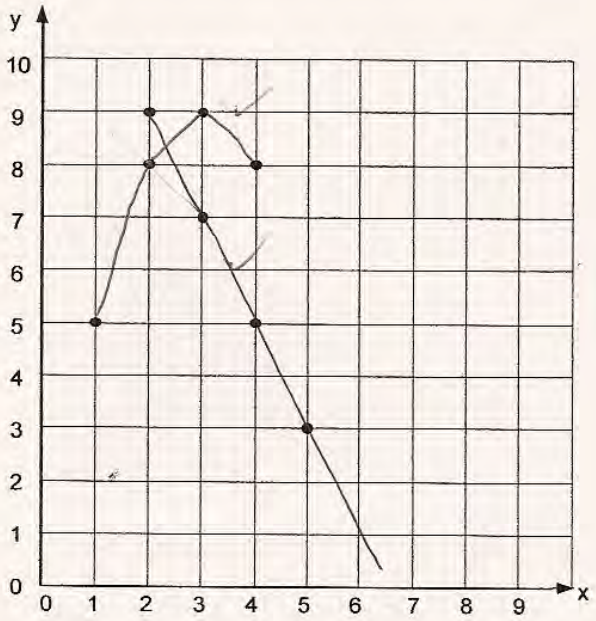
$9 - 16 = -7$

Student D is able to use slope to find the formula for part 2. In part 5 the student makes an attempt to use the x-intercepts or roots to find the equation. Due to inaccuracy in the graph the student's formula is a little off. *What do you need to know to find the correct roots? What could help the student see that the roots are incorrect?*

Student D

On the grid are eight points from two different functions.

- four points fit a **linear** function
- the other four points fit a **non-linear** function.



For the **linear** function:

1. Write the coordinate pairs of its four points.

(2,9) ✓ ✓
(3,7) ✓ ✓
(4,5) ✓ ✓
(5,3) ✓ ✓

Draw the line on the grid. ✓ 1

2. Write an equation for the function.
Show your work.

13 − (13,0)
−2
−11
(1,11)

11
−2
9
(2,9)

9
−2
−7
(3,7) ect...

$y = -2x + 13$ ✓ ✓ 2 2

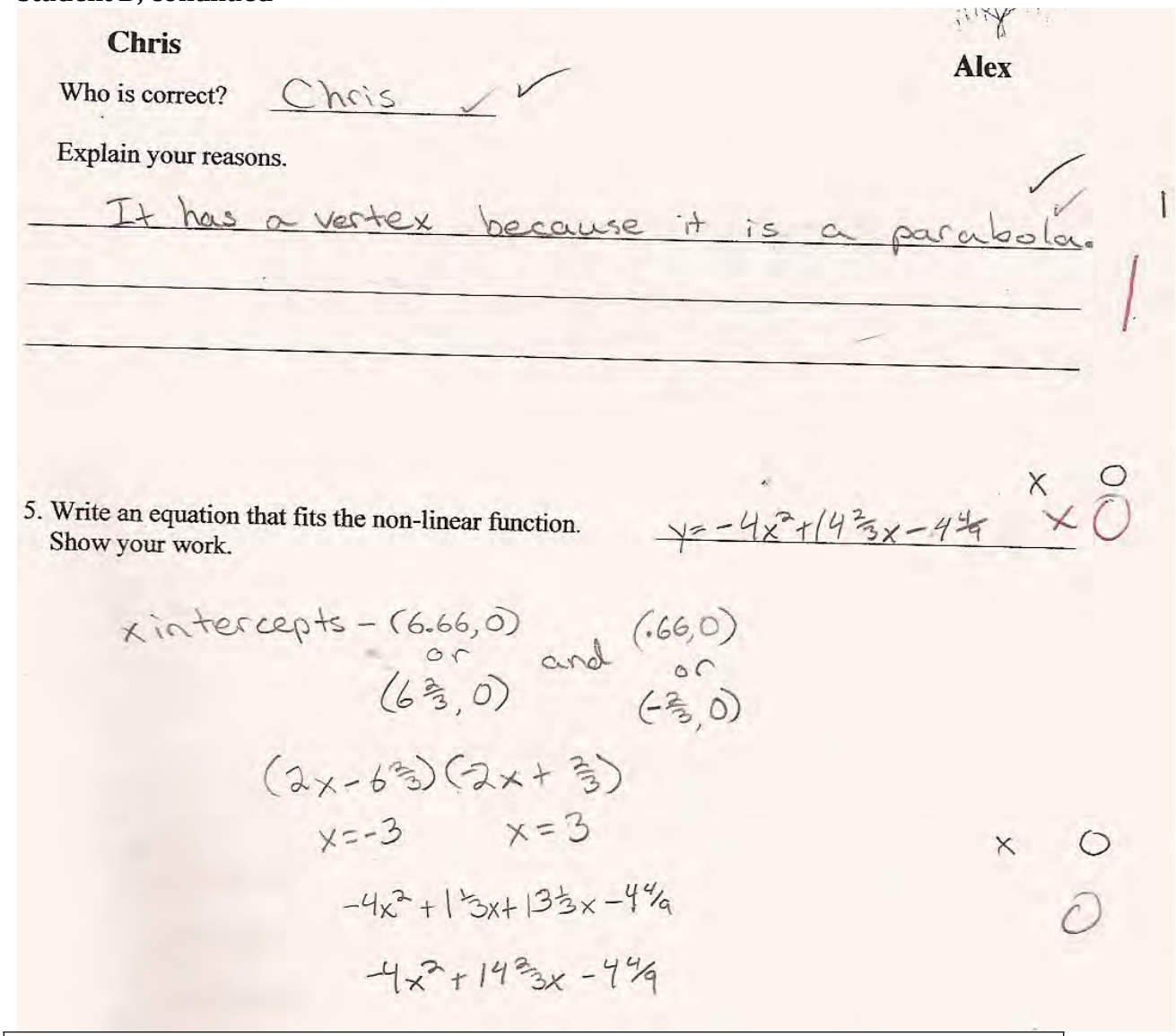
For the **non-linear** function:

3. Write the coordinate pairs of its four points.

(1,5) ✓ ✓ (2,8) ✓ ✓ (3,9) ✓ ✓ (4,8) ✓ ✓ 1 1

Draw the graph of the function on the grid. ✓ ✗ 0 1

Student D, continued



Chris

Alex

Who is correct? Chris ✓ ✓

Explain your reasons.

It has a vertex because it is a parabola. ✓ ✓ 1

1

5. Write an equation that fits the non-linear function. Show your work.

$y = -4x^2 + 14\frac{2}{3}x - 4\frac{4}{9}$ × 0

x intercepts – (6.66, 0) or $(6\frac{2}{3}, 0)$ and (.66, 0) or $(-\frac{2}{3}, 0)$

$(2x - 6\frac{2}{3})(2x + \frac{2}{3})$

$x = -3 \qquad x = 3$

$-4x^2 + 1\frac{1}{3}x + 13\frac{1}{3}x - 4\frac{4}{9}$

$-4x^2 + 14\frac{2}{3}x - 4\frac{4}{9}$

× 0

0

Student E has an interesting strategy for finding the equation in part 2. *How would you explain the strategy and why it works?* In part 5 the strategy is not so successful. *Why doesn't the strategy work this time? Could it be adjusted to fit this new situation?* Student E makes the common error of connecting exponents with exponential function.

Student E

2. Write an equation for the function.
Show your work.

$y = -2x + 13$ ✓✓

$3 = -2(5) + b$

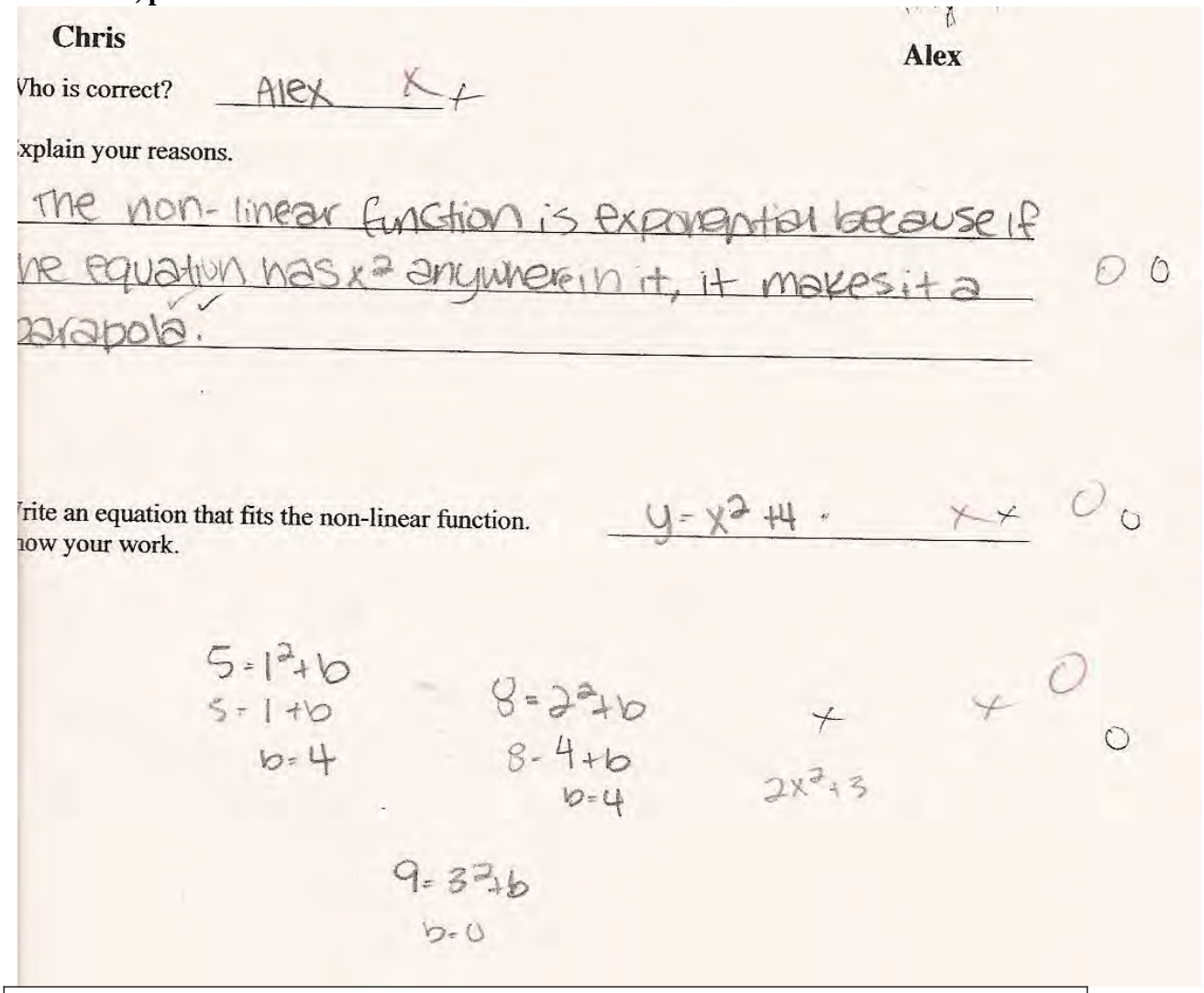
$3 = -10 + b$

$+10 \quad +10$

$13 = b$

2

Student E, part 2



Chris

Alex

Who is correct? Alex

Explain your reasons.

The non-linear function is exponential because if the equation has x^2 anywhere in it, it makes it a parabola.

Write an equation that fits the non-linear function. Show your work.

$y = x^2 + 4$

$5 = 1^2 + b$
$5 = 1 + b$
$b = 4$

$8 = 2^2 + b$
$8 = 4 + b$
$b = 4$

$2x^2 + 3$

$9 = 3^2 + b$
$b = 0$

Student F tries to use the slope to find the equation in part 5. *How would you describe the misconception of the student? Why is this equation incorrect? What question could you pose to help the student see his misconception?*

Student F

5. Write an equation that fits the non-linear function. Show your work.

$-x+12=y$ X

(3,9) (4,8)

$\frac{8-9}{4-3} = \frac{-1}{1}$ $m=-1$

$-1 = \frac{y-8}{x-4}$

$-x+4 = y-8$

$+8 \quad +8$

$y = -x+12$

Student G is able to complete correctly parts 1,2, and 3 of the task. In part four the student seems to see quadratic and exponential as describing the same function. In part 4 the student doesn't think about the writing an equation for the parabola in the first graph, but gives a general example complete with graph.

Student G

Alex

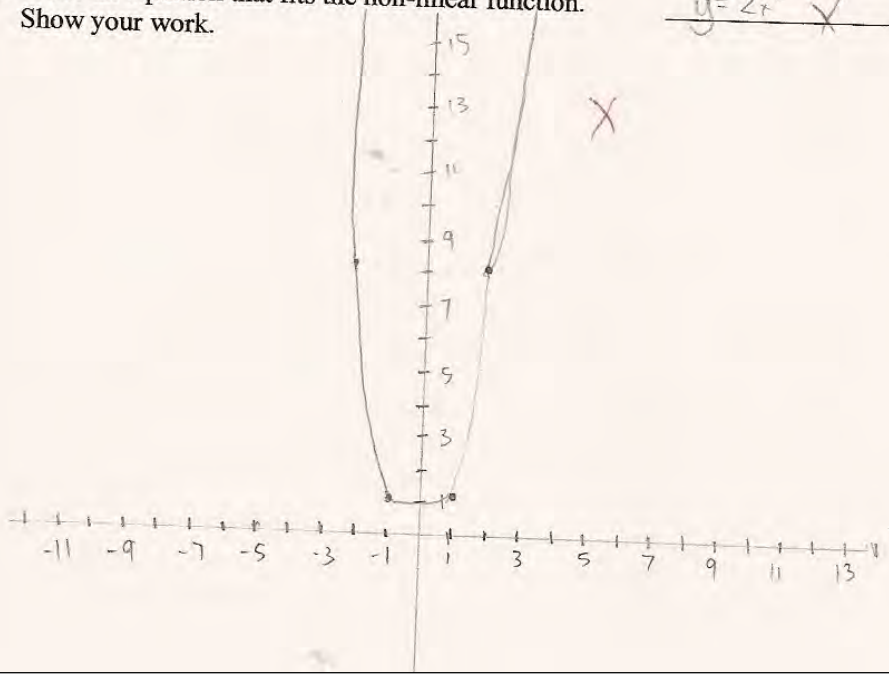
Who is correct? Both

Explain your reasons.

A non-linear function is exponential because it would not be a line but a parabola. It would also be quadratic because you would have to factor the expression and then get the coordinates for the parabola.

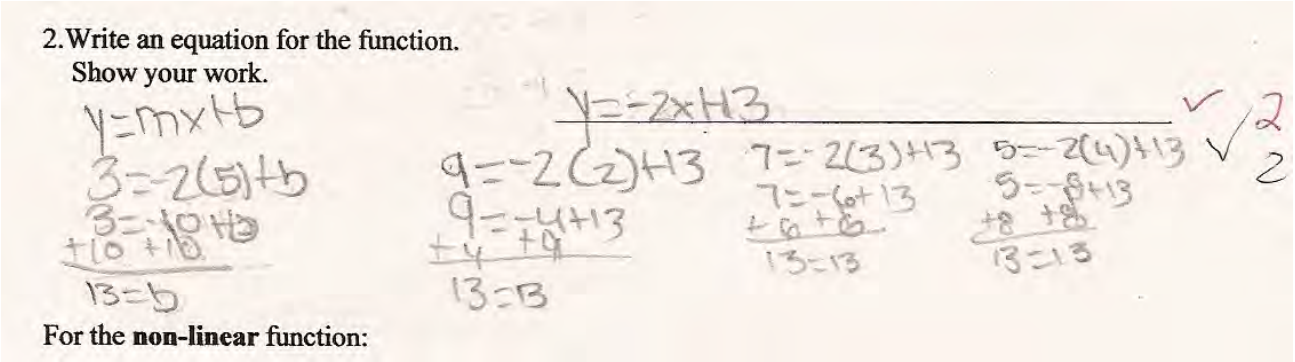
5. Write an equation that fits the non-linear function. Show your work.

$y = 2x^2$



Many students tried to use something about slope or substitution to find the formula for the quadratic. Student H uses the y=mx +b for all the points to find the constant in part 2. In part 4 the student states that quadratic equations can be factored. So the student tries the same strategy for the linear equation to find the constant in the quadratic. Notice the guess and check show factored expressions but the final answer is a linear equation. *What might be next steps for this student?*

Student H



2. Write an equation for the function.
Show your work.

$y = mx + b$

$3 = -2(5) + b$

$3 = -10 + b$

$+10 \quad +10$

$13 = b$

$y = -2x + 13$ ✓

$9 = -2(2) + 13$

$9 = -4 + 13$

$+4 \quad +4$

$13 = 13$

$7 = -2(3) + 13$

$7 = -6 + 13$

$+6 \quad +6$

$13 = 13$

$5 = -2(4) + 13$

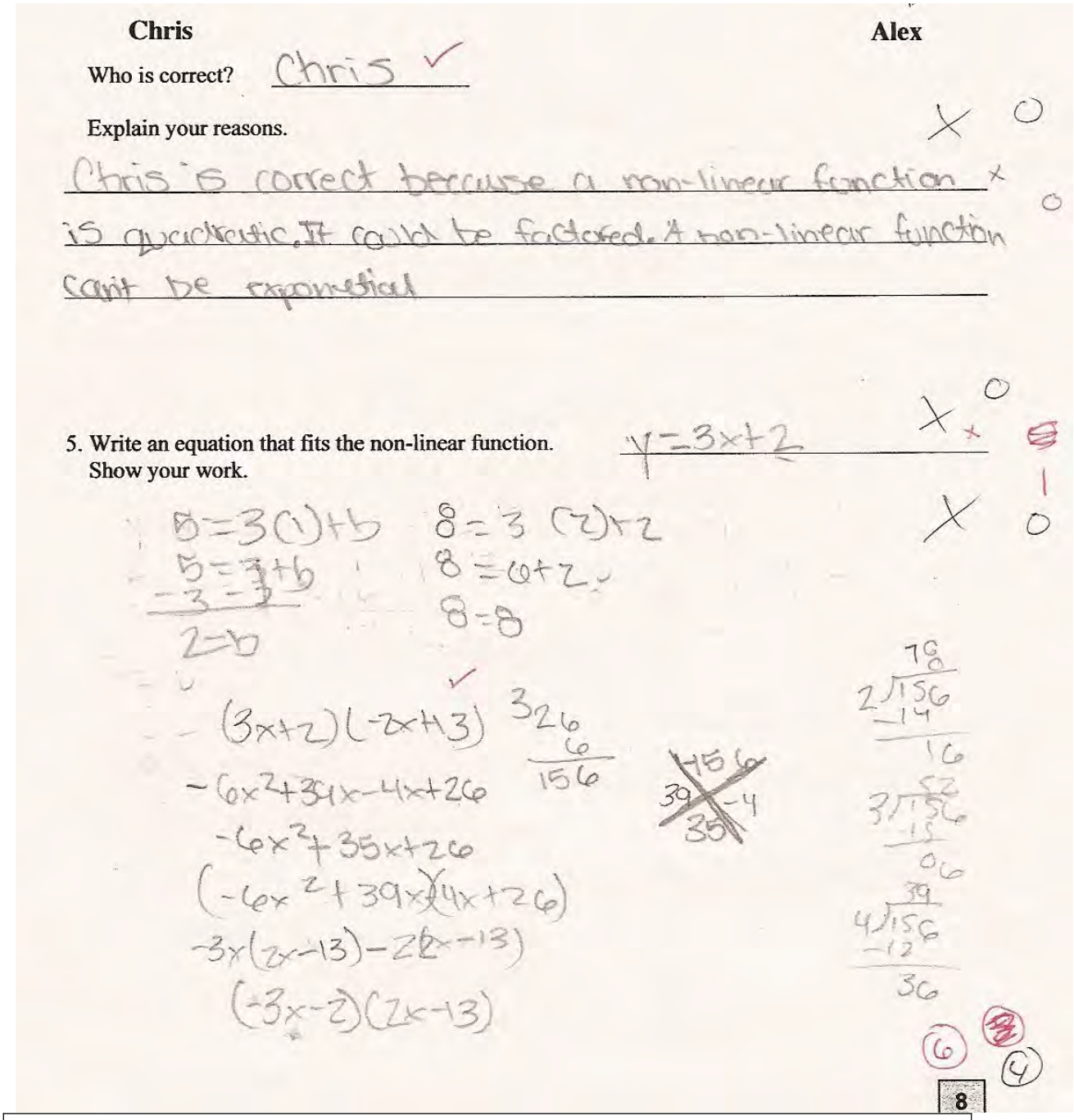
$5 = -8 + 13$

$+8 \quad +8$

$13 = 13$ ✓ 2

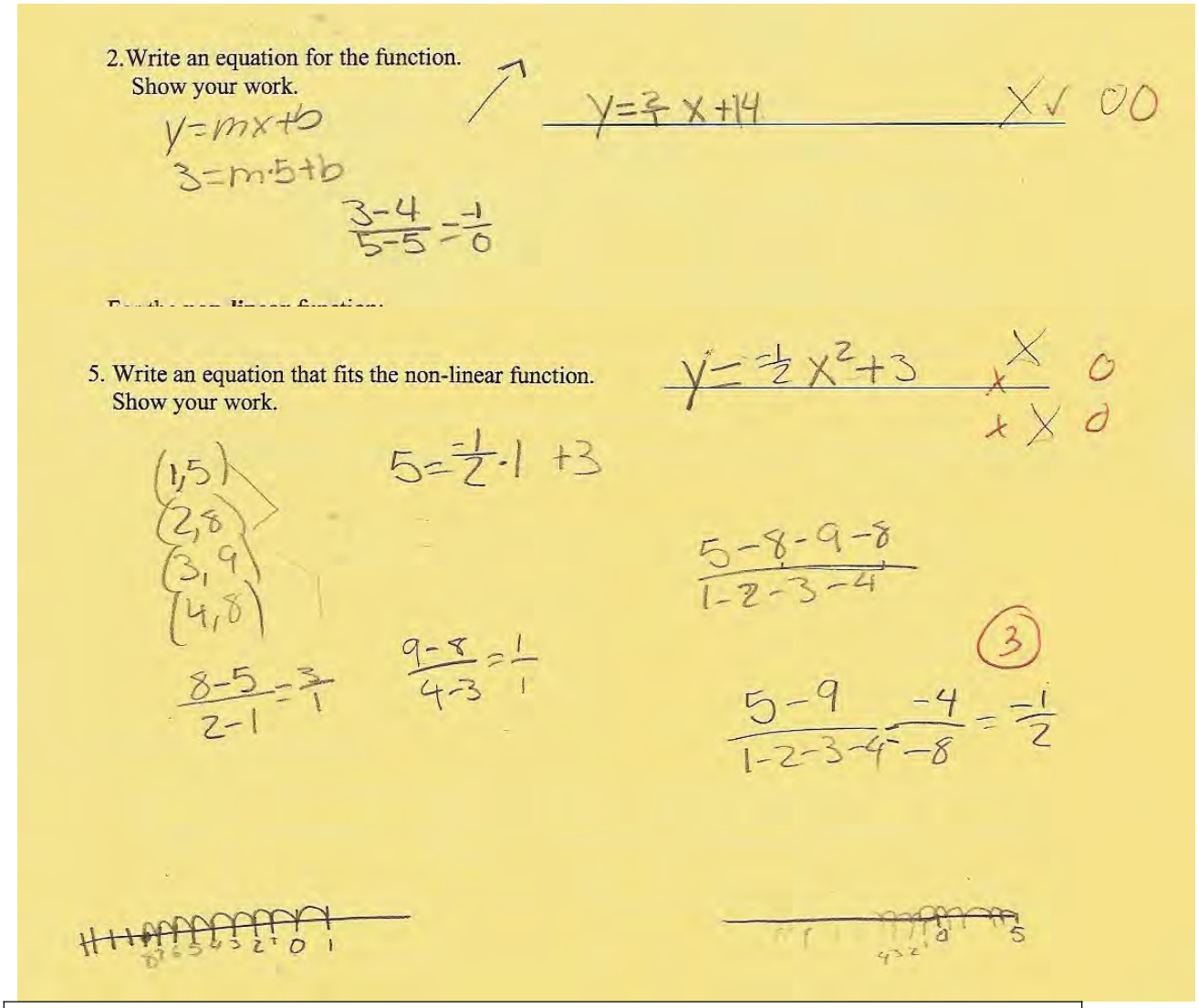
2

For the **non-linear** function:



Student I tries similar ideas, but makes an error in part 2. In part three the student just puts in all the points to try to nd the slope. Notice that the student seems to use the number line to think about combining positive and negative numbers.

Student I

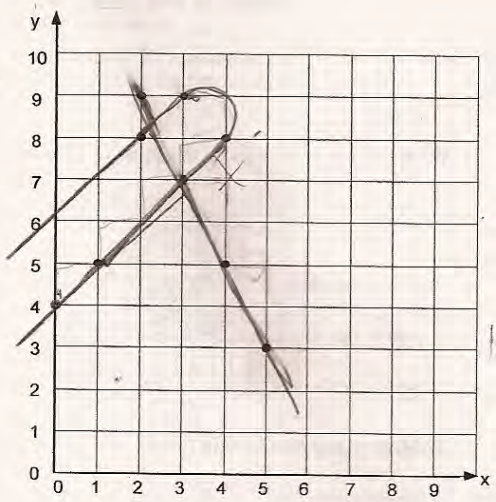


Student J is able to pick out the correct points in part 3 but draws the parabola at an angle. This leads to confusion on part 4. The student does have the habit of mind to test out formulas and check them. Notice all the work in part 5.*How could you help the student investigate quadratics more? How could you pose a whole class investigation to explore the idea of how parts of the equation e ect the shape of the graph?*

Student J

On the grid are eight points from two different functions.

- four points fit a **linear** function
- the other four points fit a **non-linear** function.



For the **linear** function:

1. Write the coordinate pairs of its four points.

(5,3)

(4,5)

(3,7)

(2,9)

Draw the line on the grid.

2. Write an equation for the function.
Show your work.

$y = -2/1x + 12$

Chris **Alex**

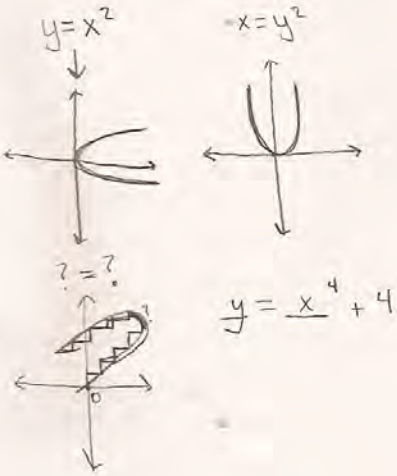
Who is correct? Alex X

Explain your reasons.

Alex is right because $y=x^2$ and $x=y^2$ are both parabolas or non-linear. The one on the graph is non-linear but still odd. These parabolas need to have an exponent of 2 or higher to get a curved angle in either of the 4 quadrants. 0/0

5. Write an equation that fits the non-linear function. Show your work.

$y = x^4 + 4$ X 0/0



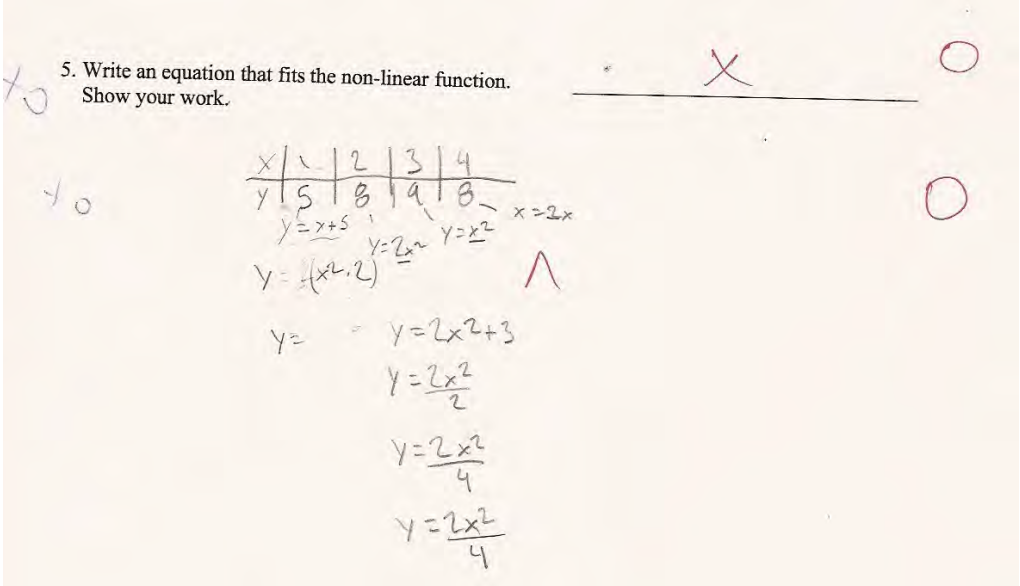
Student K attempts to find the formula in part 5 by looking at the table of values. The student finds a different formula for each set of points on the graph but doesn't know where to go from there. *What question might you pose to push the students thinking?*

Student K

2. Write an equation for the function. Show your work.

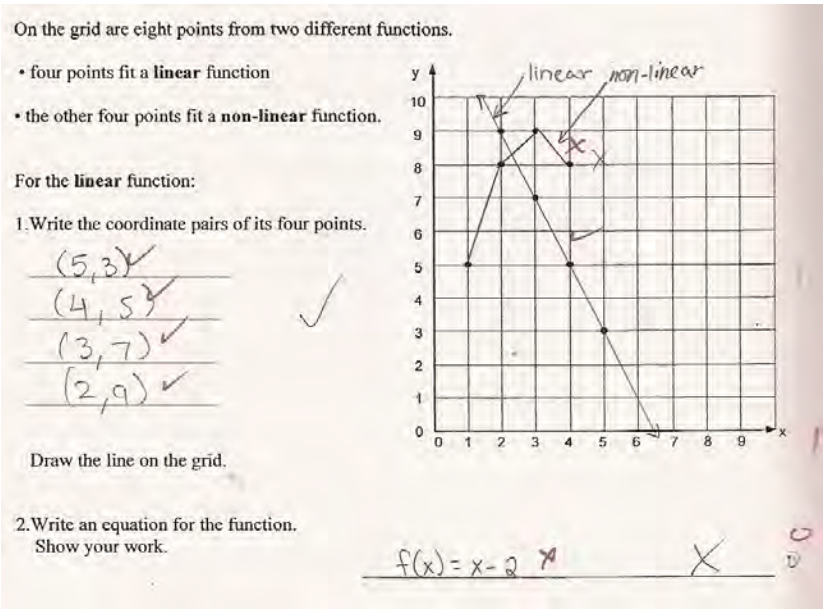
$y = 2x + 3x$ 0

X 0

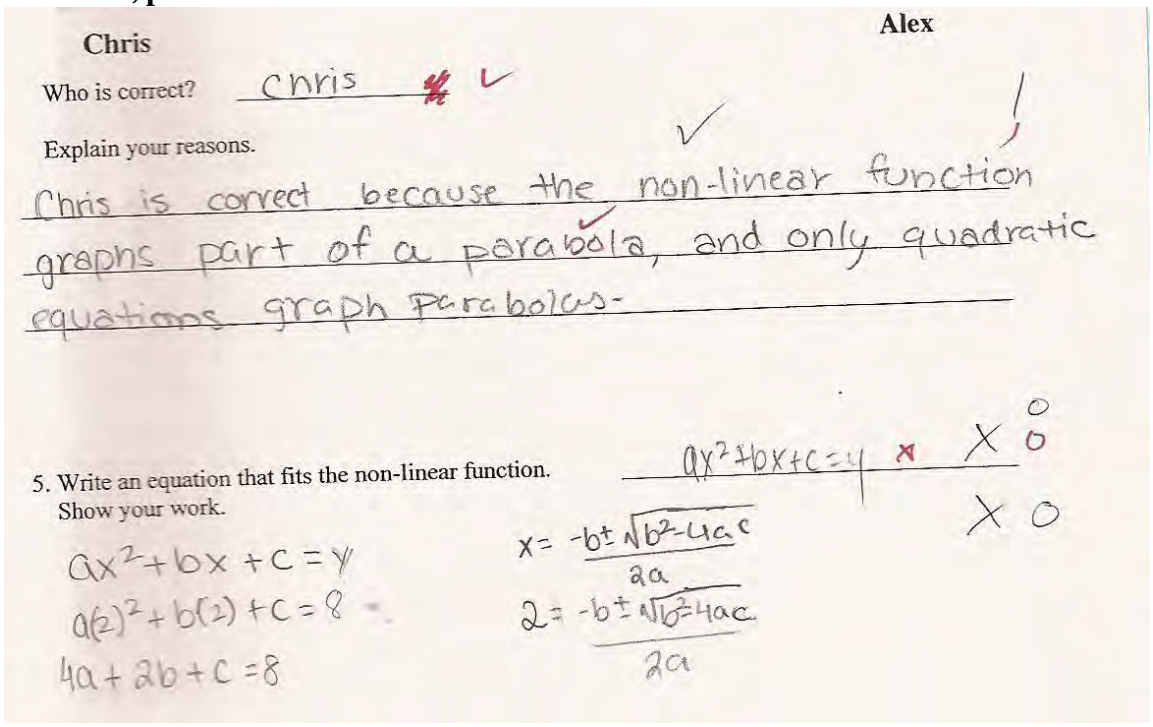


Student L understands the general equation for a quadratic and attempts to substitute points into the equation. The student does not know how to solve from there. *Is there a way of building on this idea to get a solution?*

Student L



Student L, part 2



Chris

Alex

Who is correct? chris

Explain your reasons.

Chris is correct because the non-linear function graphs part of a parabola, and only quadratic equations graph Parabolas.

5. Write an equation that fits the non-linear function. Show your work.

$ax^2+bx+c=y$

$a(2)^2+b(2)+c=8$

$4a+2b+c=8$

$ax^2+bx+c=y$ x

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Algebra Task 5 Functions

Student Task	Work with graphs and equations of linear and non-linear functions.
Core Idea 1 Functions and Relations	**Understand patterns, relations, and functions.** • Understand relations and functions and select, convert flexibly among, and use various representations for them.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Use symbolic algebra to represent and explain mathematical relationships. • Judge the meaning, utility, and reasonableness of results of symbolic manipulation.

The mathematics of this task:

- Identify linear points on a coordinate grid and name them
- Write an equation for a linear function from a graph or from coordinates
- Recognize non-linear points that form a parabola and estimate the graph of the curve
- Distinguish between features of a linear, quadratic and exponential graph and their equations
- Find the equation for a parabola given some of the coordinate points

Based on teacher observations, this is what algebra students knew and were able to do:

- Understand that a linear graph is a straight line
- Know that a non-linear graph is a parabola
- Identifying points on a graph

Areas of difficulty for algebra students:

- Finding a linear equation from a graph
- Finding a quadratic equation
- Drawing a parabola
- Difficulty in knowing difference between quadratic and quadrilateral (four points/four sides)
- Confusion about quadratic and exponential equations(x^2 has an exponent so it was exponential)

Strategies used by successful students:

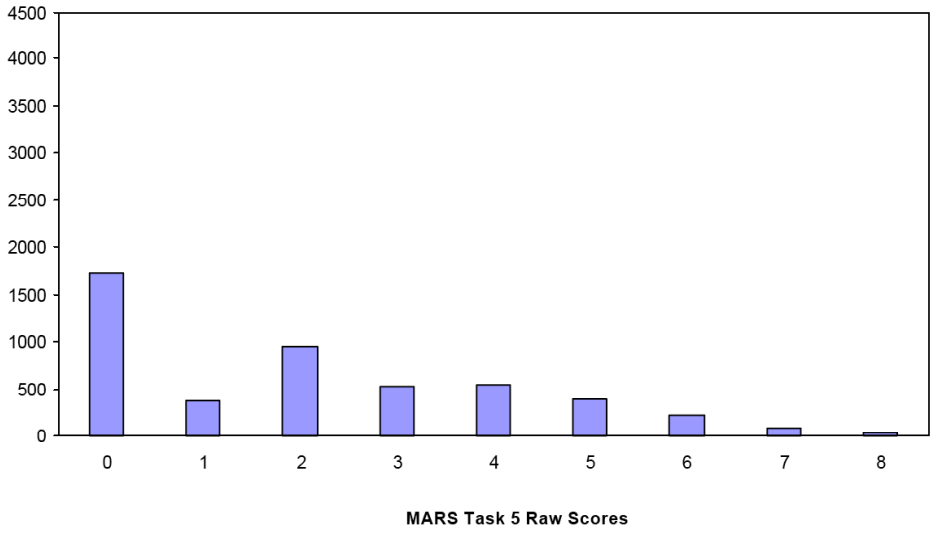
- Knowing the generic formula for a line: $y = mx + b$ and using slope and substitution
- Looking for the y-intercept for the linear equation
- Finding slope
- Knowing the generic formula for a quadratic: $y=a x^2+bx + c$
- Knowing mathematical vocabulary: quadratic, exponential

- Extending the parabola to find more points, particularly the roots or x intercepts

Table 49: Frequency Distribution of MARS Test Task 5, Course 1

Task 5 Scores	Student Count	% at or below	% at or above
0	1737	35.6%	100.0%
1	381	43.4%	64.4%
2	959	63.0%	56.6%
3	528	73.8%	37.0%
4	545	85.0%	26.2%
5	399	93.2%	15.0%
6	215	97.6%	6.8%
7	79	99.2%	2.4%
8	39	100.0%	0.8%

Figure 58: Bar Graph of MARS Test Task 5 Raw Scores, Course 1



The maximum score available for this task is 8 points.
The minimum for a level 3 response, meeting standards, is 4 points.

Some students, about 64%, could correctly identify, name and graph the four linear points. About 56% of the students could also name the remaining four non-linear points. About 37% could also graph the curve of the parabola. Only 26% could also give an equation for the linear function. Only 7% could explain that the parabola was a quadratic equation. Less than one percent of the students could meet all the demands of the task including giving a equation for the parabola. Almost 36% of the students scored no points on this task. About 65% of the students with this score attempted the task.

Functions

Points	Understandings	Misunderstandings
0	65% of the students with this score attempted the task.	Students usually knew how to order coordinate points but misread one of the points or put (2,8) for (2,9). Less than 3% choose all the wrong points. Less than 2% reversed the x- and y-coordinates.
1	Students could name points on a coordinate graph, identify the ones that were linear and draw the graph.	About 6% misnamed one of the coordinate pairs for the parabola. About 9% did not attempt to name the non-linear points.
3	Students could name the linear and non-linear points and graph a straight line and a parabola.	Almost 20% of the students made parabolas that were 4 line segments. 12% of the students did not attempt to graph these points.
4	Students could name and graph points in a linear function, give the equation of the line, and name the non-linear points.	17% of the students did not attempt to write an equation for the line. About 12% of the students had equations with –2x but the incorrect constant. About 5% just wrote y=mx +b. 5% gave numeric expressions or discrete values for x and y.
5	Students could identify and graph points for linear functions and quadratics. They could write equations for a linear function from the graph.	Students could not distinguish between quadratic and exponential equations.25% of the students did not attempt part 4. About 12% knew that parabolas have exponents, so it should be exponential. About 5 % confused quadratic with 4 sides. About 4% thought both words were the same.
6	Students could distinguish between quadratic and exponential functions.	Students struggled with the equation for the parabola. 36% of the students did not attempt this part of the task. About 5% attempted to look at a table of values, but couldn't figure out what to do with the information. About 3% knew the general formula, $ax^2+bx+c=y$, but couldn't figure out what to do next. About 10% wrote linear equations. About 12% wrote incorrect equations with exponents.
8	Students could identify and graph linear and quadratic points and give their equations.	

Implications for Instruction

Students at this grade level should be able to make connections between equations and the shape of a graph. Students should be able to recognize proportional functions ($y = mx$) with linear equations going through the origin. Students should also be able to explain how the constant, the added portion of the equation $y=mx+b$, affects the graph by raising or lowering the equation without changing the slope and also indicates the y-intercept. Students should also understand that equations with exponents will not be linear. The most common form $y=mx^2+bx + c$ will form a parabola. Students should be able to make predictions about the equation from looking at a graph or predictions about the graph from looking at the equation. Some students will start to move beyond this to think about quadratic equations and exponential equations or be able to write an equation for parabolas, but that is the ramp of this task. Students should be familiar with multiple representations and knowing how they connect to each other. Students should know how the parts of the equation relate to the shape of the graph knowing general form of the equations

Action Research – Investigations - Linking Equations to Graphs

Relationships cannot be given to students, but must be discovered through practice and exploration. In order to learn to think like a mathematician students at this grade level need opportunities to investigate relationships. They need big rich tasks that give them opportunities to work with longer reasoning chains, learn to organize information, make systematic changes to observe the effects, and start to think about types of numbers and how they might affect outcomes. Exploring this relationship between graphs and their equations is an excellent context for investigation.

Pose a dilemma or question. For example:

I overheard two students in another class discussing graphs.
Barbara said, "All equations without exponents make straight lines." Fred added, "And they also go through zero." Don disagreed. He said that some equations without exponents don't go through zero and some are not lines." Who do you think is correct? How can you tell just by looking at an equation what a graph will look like? Can you tell how each part of the equation contributes to the shape of the graph? Organize some information that would help Barbara, Fred, and Don correct their thinking.

After students have worked with this prompt, they should make posters to share with the class or make some equations that they think might stump their classmates.

A further challenge might be to look at equations with exponents. The new prompt might be:

Find an equation that will go through (0,0) and (0,-1).

To find this solution some students will be able to apply knowledge from factoring equations. Others may be led into a rich investigation in how the numbers in equations with exponents effect the shape of the graphs.

In learning a new field of study, like algebra, students need to acquire vocabulary and procedural knowledge. But students also need the challenge and cognitive demand to work within the discipline to solve problems and make discoveries. They need to view the procedures and knowledge they are gaining as tools to make and test conjectures. It is the richness of exploring ideas that makes the mathematics of algebra interesting and engaging for students. It is the personal aha's that make the learning satisfying and personal.

Reflecting on the Results for Algebra as a Whole:

Think about student work through the collection of tasks and the implications for instruction. What are some of the big misconceptions or difficulties that really hit home for you?

If you were to describe one or two big ideas to take away and use for planning for next year, what would they be?

What are some of the qualities that you saw in good work or strategies used by good students that you would like to help other students develop?

Four areas that stand out for the Collaborative as a whole for Algebra are:

- Moving Between Multiple Representations - Students had difficulty moving from graphs to equations. Students did not recognize how the structure of the equation would effect features of the graph. This was evident in both Functions and Sorting Functions.