

I. Warm-up (5 – 10 minutes)

Present the students with 3 or 4 Geometric sequences. Ask questions appropriate to whether or not the students have studied Geometric sequences and series. For instance:

3, 6, 12, 24, 48.....

(Possible questions)

Find the next three terms in the sequence...What is the common ratio?...Write an equation that describes what is happening in the sequence...Write the sequence in summation notation...Find the 18<sup>th</sup> term

*(optional – casually introduce the term “multiplier” as an alternative to “common ratio”)*

II. Introduction (10 – 15 minutes)

Split students into groups or pairs and ask them to work on the activity “*One Grain of Rice*” from NCTM’s “*Illuminations*”. The activity consists of two parts: (1) Reading the story (which is attached and also available online @

<http://jwilson.coe.uga.edu/EMT668/EMAT6680.F99/Martin/instructional%20unit/day4.exponential/excel/grainofrice.html> and the worksheet, which is attached and available online @ <http://illuminations.nctm.org/Lessons/OneGrainRice/OneGrainRice-AS-OGR.pdf>

This activity is being used as an introduction, so the instructor is encouraged to help the students and/or bring the class together as a whole for brief instruction as needed to help the students get through the problem. They should be allowed to struggle a little – but not to the point of frustration! One suggested option is to print only the “front” page (the exploration) and lead the class as a whole in the process of finding mathematical expressions that fit the situation.

*Note --- the amount of time spent on the “One Grain of Rice” problem and warm-up (above) determine whether this lesson will be a 2-day lesson or a 3-day lesson. Teachers interested in using only 2 days may want to pare it down or remove that component altogether.*

III. Main Tasks (25 – 35 min)

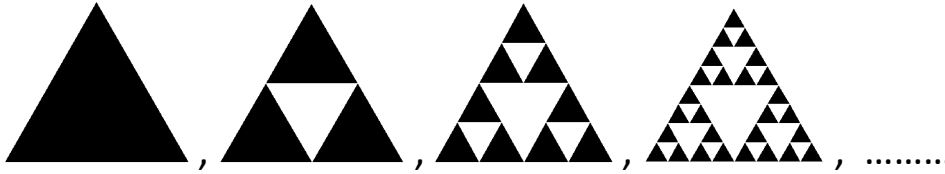
Allow students to get back into groups/pairs and work through the tasks that follow. The instructor is encouraged to look for opportunities to relate the student’s experience back to their work with Geometric /Arithmetic sequences.

Population Explosion! (simple exponential growth)

Saving for a Car (compound interest)

### Warm Up

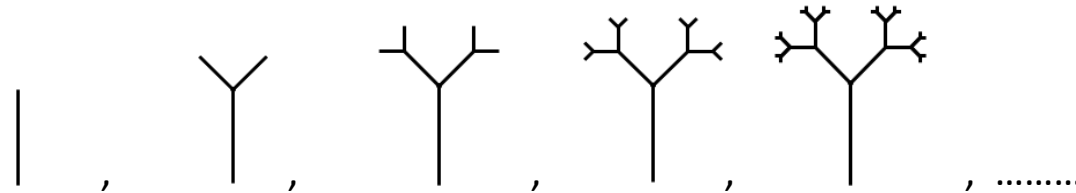
1. Consider the following sequence of patterns:



$T_1$	$T_2$	$T_3$	$T_4$
Number of triangles	Number of triangles	Number of triangles	Number of triangles

- a. How many shaded triangles do you see in each step?
- b. How many shaded triangles would be in  $T_5$ ?
- c. How many shaded triangles would be in  $T_8$ ?
- d. How many shaded triangles would be in  $T_{21}$ ?

2. Consider the following sequence of patterns:



$B_1$	$B_2$	$B_3$	$B_4$	$B_5$
Number of segments	Number of segments	Number of segments	Number of segments	Number of segments

- a. How many segments do you see in each step?
- b. How many segments would be in  $B_6$ ?
- c. How many segments would be in  $B_9$ ?
- d. \*\*How many segments would be in  $B_{32}$ ?

## Part One: ***One Grain of Rice*** a mathematical folktale by Demi

Long ago in India, there lived a raja who believed he was wise and fair, as a raja should be. The people in his province were rice farmers. The raja decreed that everyone must give nearly all of their rice to him. "I will store the rice safely," the raja promised the people, "so that in time of famine, everyone will have rice to eat, and no one will go hungry." Each year, the raja's rice collectors gathered nearly all of the people's rice and carried it away to the royal storehouses.

For many years, the rice grew well. The people gave nearly all of their rice to the raja, and the storehouses were always full. But the people were left with only enough rice to get by. Then one year the rice grew badly and there was famine and hunger. The people had no rice to give to the raja, and they had no rice to eat. The raja's ministers implored him, "Your highness, let us open the royal storehouses and give the rice to the people, as you promised." "No!" cried the raja. How do I know how long the famine will last? I must have the rice for myself. Promise or no promise, a raja must not go hungry!"

Time went on, and the people grew more and more hungry. But the raja would not give out the rice. One day, the raja ordered a feast for himself and his court--as, it seemed to him, a raja should now and then, even when there is famine. A servant led an elephant from a royal storehouse to the palace, carrying two full baskets of rice. A village girl named Rani saw that a trickle of rice was falling from one of the baskets. Quickly she jumped up and walked along beside the elephant, catching the falling rice in her skirt. She was clever, and she began to make a plan.

At the palace, a guard cried, "Halt, thief! Where are you going with that rice?"

"I am not a thief," Rani replied. "This rice fell from one of the baskets, and I am returning it now to the raja."

When the raja heard about Rani's good deed, he asked his ministers to bring her before him.

"I wish to reward you for returning what belongs to me," the raja said to Rani. "Ask me for anything, and you shall have it."

"Your highness," said Rani, "I do not deserve any reward at all. But if you wish, you may give me one grain of rice."

"Only one grain of rice?" exclaimed the raja. "Surely you will allow me to reward you more plentifully, as a raja should."

"Very well," said Rani. "If it pleased Your Highness, you may reward me in this way. Today, you will give me a single grain of rice. Then, each day for thirty days you will give me double the rice you gave me the day before. Thus, tomorrow you will give me two grains of rice, the next day four grains of rice, and so on for thirty day."

"This seems to be a modest reward," said the raja. "But you shall have it."

And Rani was presented with a single grain of rice.

Now, it is the student's job to build a spreadsheet based upon this story to determine the amount of rice given to Rani on any given day. We start the spreadsheet by listing under the heading, "day," 1....30. Then we proceed with the following formula:  $2 * (\text{previous cell address})$  and fill down. After each day, read the part of the story that corresponds to that day. (Read the number AFTER they find it using the spreadsheet)

The next day, Rani was presented with two grains of rice.

And the following day, Rani was presented with four grains of rice.

On the ninth day, Rani was presented with two hundred fifty-six grains of rice. She had received in all five hundred and eleven grains of rice, enough for only a small handful. "This girl is honest, but not very clever," thought the raja. "She would have gained more rice by keeping what fell into her skirt!"

On the twelfth day, Rani received two thousand and forty-eight grains of rice, about four handfuls.

On the thirteenth day, she received four thousand and ninety-six grains of rice, enough to fill a bowl.

On the sixteenth day, Rani was presented with a bag containing thirty-two thousand, seven hundred and sixty-eight grains of rice. All together she had enough rice for two bags. "This doubling up adds up to more rice than I expected" thought the raja. "But surely her reward won't amount to much more."

On the twentieth day, Rani was presented with sixteen more bags filled with rice.

On the twenty-first day, she received one million, forty-eight thousand, five hundred and seventy-six grains of rice, enough to fill a basket.

On the twenty-fourth day, Rani was presented with eight million, three hundred and eighty-eight thousand, six hundred and eight grains of rice--enough to fill eight baskets, which were carried to her by eight royal deer.

On the twenty-seventh day, thirty-two brahma bulls were needed to deliver sixty-four baskets of rice. The raja was deeply troubled. "One grain of rice has grown very great indeed," he thought. "But I shall fulfill the reward to the end, as a raja should."

On the twenty-ninth day, Rani was presented with the contents of two royal storehouses.

On the thirtieth and final day, two hundred and fifty-six elephants crossed the province, carrying the contents of the last four royal storehouses--Five hundred and thirty-six million, eight hundred and seventy thousand, nine hundred and twelve grains of rice.

All together, Rani had received more than one billion grains of rice. The raja had no more rice to give. "And what will you do with this rice," said the raja with a sigh, "now that I have none?"

"I shall give it to all the hungry people," said Rani, "and I shall leave a basket of rice for you, too, if you promise from now on to take only as much rice as you need."

"I promise," said the raja. And for the rest of his days, the raja was truly wise and fair, as a raja should be.

## One Grain of Rice

NAME \_\_\_\_\_

In the book *One Grain of Rice* by Demi, the main character Rani cleverly tricks the raja into giving rice to the village. Use the story from the book to answer the questions below.

- Estimate how many grains of rice you think Rani will have at the end of 30 days.
- Use the chart below to record the number of grains of rice Rani would receive each day.

Day 1 <b>1</b> grain of rice	Day 2 <b>2</b> grains of rice	Day 3 grains of rice	Day 4 grains of rice	Day 5 grains of rice	<b>Total After 5 Days</b>
Day 6 grains of rice	Day 7 grains of rice	Day 8 grains of rice	Day 9 grains of rice	Day 10 <b>512</b> grains of rice	<b>Total After 10 Days</b>
Day 11 grains of rice	Day 12 grains of rice	Day 13 grains of rice	Day 14 grains of rice	Day 15 grains of rice	<b>Total After 15 Days</b>
Day 16 grains of rice	Day 17 grains of rice	Day 18 <b>131,072</b> grains of rice	Day 19 grains of rice	Day 20 grains of rice	<b>Total After 20 Days</b>
Day 21 grains of rice	Day 22 grains of rice	Day 23 grains of rice	Day 24 grains of rice	Day 25 grains of rice	<b>Total After 25 Days</b>
Day 26 grains of rice	Day 27 grains of rice	Day 28 grains of rice	Day 29 grains of rice	Day 30 grains of rice	<b>Total After 30 Days</b>

- If the story continued and you know how many grains of rice Rani receives on Day 30, how can you determine how many grains of rice she would receive on Day 31?

How can you determine how many grains of rice she would receive on Day 35?

How can you determine how many grains of rice she would receive on Day 40?

If you know how many grains of rice she receives on a certain day, how can you determine how many grains of rice she will receive 2 days later? ...10 days later?

## Part Two: Population Explosion!

You live in the village of Algebraville, located on a beautiful hillside overlooking the river Pi. On the other side of the river lies your sister village Geometricus. The two villages are in constant competition, especially at the annual Pi Games (a celebration of the great mathematical accomplishments of your country --- Calculand). The two villages are both growing exponentially, but Algebraville is currently much larger than Geometricus (something they're quite proud of!) – with a population of 2,500, compared to the 1,000 people who live in Geometricus.

Assume that Algebraville's population is increasing by 30% each year. Find a single number by which you could multiply the current population in order to find out what next year's population would be (i.e. find your "multiplier"). Then write an expression which can be used to find the population in future years (let your "x" be the number of years after our starting population of 2,500).

Multiplier: \_\_\_\_\_

Expression for future population:

\_\_\_\_\_

Now, suppose that Geometricus has a population growth of 70% each year. Find a single number by which you could multiply the current population in order to find out what next year's population would be (i.e. find your "multiplier"). Then write an expression which can be used to find the population in future years and complete the chart.

Multiplier: \_\_\_\_\_

Expression for future population:

\_\_\_\_\_

Complete the chart below for the **Algebraville** population:

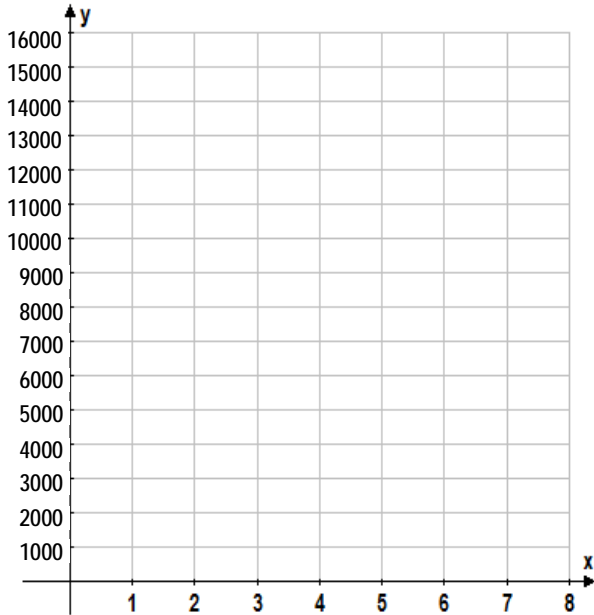
Year	Population
0	2,500
1	
2	
3	
4	
5	

Complete the chart below for the **Geometricus** population:

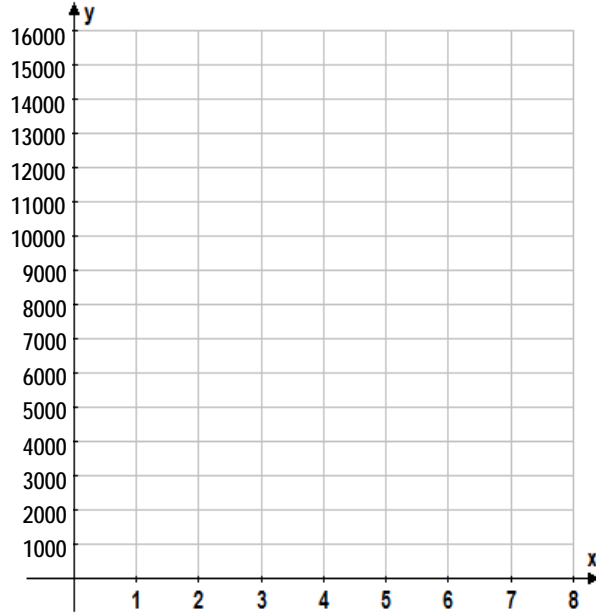
Year	Population
0	1,000
1	
2	
3	
4	
5	

Now, let's look at the situation as a graph. Make a scatter plot to represent each village's growth (by hand), then connect the points to form a smooth curve.

### Algebraville



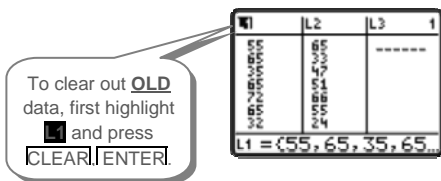
### Geometricus



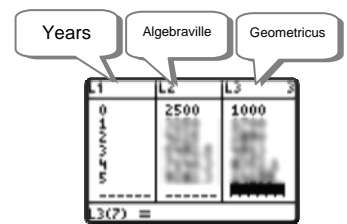
The next several steps involve using the graphing calculator to help us compare the two villages. First, let's check our work above by using the curve-fitting abilities of the graphing calculator.

1) Next, press **STAT** **ENTER**

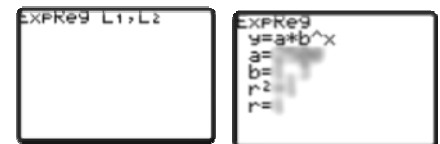
2) If there is OLD data already in the lists that needs to be cleared press the up arrow, **↑**, to highlight **L1** and then press **CLEAR** **ENTER** to clear out the old data. Do the same for L2 if it has OLD data that needs to be cleared.



3) Looking at the charts you made for each village, we're going to create three lists. For L1 enter the x-values (0,1,2,3,4,5). Now, let Algebraville be L2 and Geometricus be L3.



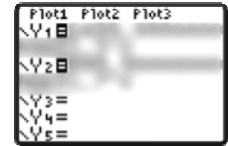
4) Return to the home screen by pressing **2nd** **MODE** and then to calculate the linear regression press **STAT** **→** **0** **2nd** **1** **,** **2nd** **2**.



Note that you will have to tell the program which lists you're using: Repeat the steps above but try comparing **EXPREG L1, L3**

Does your equation match for Algebraville? Geometricus?  
 If your equations don't match what you wrote earlier, why not?

- 5) Now, let's graph both equations together. Go to **Y=**
- 6) You can either type in your two equations in  $Y_1$  and  $Y_2$  or you can paste in the previous calculated equation by pressing **Y=** **VAR** **5** **→** **→** **1**.
- 7) Remember, before you hit **GRAPH** that you need to change your window to be able to see the graph!! First press **WINDOW**.



The window settings are related to the domain and range of each function.

What are the smallest and largest  $x$ -values for Algebraville....for Geometricus? What do the  $x$ -values signify? What does this have to do with the domain of the function?

What are the smallest and largest  $y$ -values for Algebraville....for Geometricus? What do the  $x$ -values signify? What does this have to do with the range of the function?

Compare the minimum  $x$ - and  $y$ -values to determine an appropriate Window for graphing BOTH equations on one graph (fill in the blanks below)

XMIN: \_\_\_\_\_

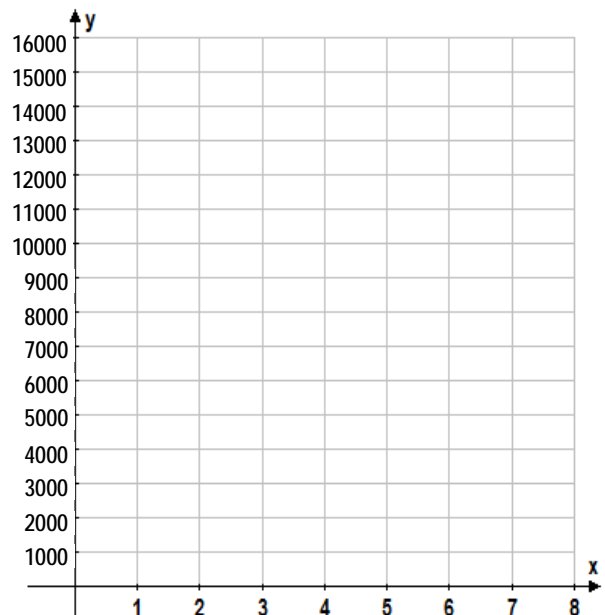
XMAX: \_\_\_\_\_

YMIN: \_\_\_\_\_

YMAX: \_\_\_\_\_

Sketch your graph below, then shade your graph with two colors as described.

- Shade with one color the space that shows when Algebraville's population will be greater than Geometricus'.
- Shade with a second color the space that shows when Geometricus' population will be greater than Algebraville's.
- Finally, use your pen or pencil to circle the place on the graph that shows when the two villages will have the same population (boy, the Pi Games will be exciting that year!).



Use any method to determine how many years from now

this will occur: \_\_\_\_\_ Can you determine in which month it might occur? \_\_\_\_\_





- 3) Using  $A = P\left(1 + \frac{r}{n}\right)^{nt}$  where  $A$  is the amount earned at the end of  $t$  years,  $P$  is the initial investment,  $r$  is the interest rate,  $n$  is the number of times compounded in 1 year, and  $t$  is the number of years of investment, found out how much money each plan will provide.
- 4) What was the optimal plan and how much money did it return?

On Mark's 15th birthday, his parents gave him another choice:

- A. 32 MB iPod with \$100 iTunes gift card now and a Nikon Coolpix digital camera for Christmas, OR
  - B. Invest the \$525 plus the best returns from the previous CD into another 1-year CD.
- 5) What choice would you choose?
- 6) Taking the optimal plan from Question #4, Mark's parents reinvested the money returned plus \$525 into a new CD with the same compounded interest. How much money will Mark have to purchase a used car when he turns 16?